PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 70 APRIL, 1944 No. 4, PART 1

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

Printed in the United States of America

CONTENTS FOR APRIL, 1944 (*Continued*)

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK

By Yves Nubar,[1] M. Am. Soc. C. E.

Synopsis

A method is presented for the analysis of displacements and stresses in the general two-dimensional framework composed of rectilinear members, subjected to loading and displacement in its own plane. The joints may be articulated or rigid, or they may be a combination of such types.

The analysis is characterized by the division or decomposition of the frame into triangular elements by means of imaginary diagonals and the consideration of the axial deformations of the bars (given or imaginary) of the resulting network.

The technical literature contains descriptions of many excellent methods for the analysis of frames, some of which deal with specialized types. Most of them can be loosely classified as being based on one of the following concepts: Least work, slope deflection, fixed points, moment distribution, angle distribution, etc. The method developed in this paper has a semi-geometric basis. It is applicable to simple structures, as well as to complex systems of straight lines meeting at joints to form panels of arbitrary shape. Its value consists perhaps in the development of certain inherent properties of the general framework, helpful in the systematization of the analysis. This tends to unify, under one consistent procedure, the method of approach in a number of superficially unrelated problems, such as, for instance, the determination of deflections in a hinged truss and of moments in a rigid frame.

The application of the method is illustrated by several practical cases.

Introduction

General Description.—In this analysis, the frame is first decomposed completely into triangular elements by means of imaginary diagonals, in an arbi-

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 1, 1944.

[1] Cons. Engr., New York, N. Y.

trary but simple fashion (see dotted lines in Fig. 1). These diagonals may be conceived as infinitely thin wires stretched between the joints, carrying infinitesimal forces, but undergoing finite axial strains as a result of the deformation of the frame. The axial strains of the given members and those of the imaginary diagonals are taken as the unknowns, with the understanding that the axial strains in given or imaginary members joining supports are equivalently written in terms of the linear movements of the supports. These elements determine the relative positions of all deflected joints and supports and lead to the solution of stresses. The number of these unknowns is greatly reduced in the case (frequent in problems on rigid frames) in which the axial deformations of the given members are assumed to be negligible and the linear movements of the supports are prevented.

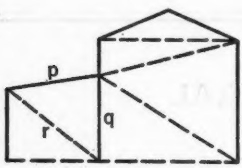


FIG. 1

The following is an outline of the procedure of the analysis:

A.—Since closed circuits must remain so after deformation, conditions of compatibility must be satisfied by the axial and lateral strains of the sides of all closed circuits. For triangular elements such as $p\ q\ r$ of Fig. 1, these conditions are written in a form involving the axial strains of the three sides and the difference $\delta_p - \delta_q$ (expressed as δ_{p-q} in this paper) between the lateral strains δ_p and δ_q of any two of them. The axial strains, adopted as unknowns in this analysis, may be interdependent; this follows from restrictions imposed by the compatibility conditions. These conditions lead to the following results:

(*a*) Expression of elements δ_{p-q} in terms of the unknowns;

(*b*) Determination of the relative displacement of any two joints of the frame in terms of the unknowns; and

(*c*) Reduction of the unknowns to a minimum number of them which are independent.

B.—Relationships between the dependent and independent unknowns are developed through a study of the topological properties of the general frame. (Topology is defined by R. Courant and H. E. Robbins[2] as "the study of the properties of geometrical figures that persist even when the figures are subjected to deformations so drastic that all their metric and projective properties are lost." In a framework, the numbers of members, joints, regions, etc., are thus topological elements, and relations between them represent topological properties of the framework.) Such a study discloses the degree of freedom of the frame in joint displacements and relates it to the number of independent axial strains.

C.—The boundary conditions are translated into relations which exclude the support reactions and involve the unknowns of this analysis.

D.—All end moments (and consequently tranverse end forces) are expressed, by means of recurrence formulas, in terms of δ_{p-q}; that is, in terms of the independent unknowns (see A(*a*) and A(*c*)).

[2] "What is Mathematics?" by Richard Courant and Herbert E. Robbins, Oxford Univ. Press, London and New York, p. 235.

E.—The independent unknowns are determined. This results from the consideration of a conjugate frame obtained by removing all bars corresponding to dependent unknowns (see A(c)) and substituting hinges for all the joints. This conjugate frame is statically determinate; its solution yields the exact number of conditions for the determination of the independent unknowns. In general, the number of these unknowns is rather small; for instance, in the case of a one-story bent of n columns on rigid supports, with the axial deformations of given members assumed to be negligible, this number is 1.

F.—All stresses and joint displacements are now found using the values of the unknowns just determined.

Notation

The letter symbols in this paper are defined when they first appear and are assembled for convenience of reference in the Appendix.

Assumptions

All assumptions leading to the validity of the well-known equation,

$$\frac{d^2y}{dx^2} = \mp \frac{M}{E\,I} \qquad (1)$$

which relates displacements to applied loads, are adopted in this analysis. (In Eq. 1, x and y are the coordinates of a point on the deflected member, M is the bending moment, E the Young modulus, and I the moment of inertia—all referred to this point.) In particular, the assumption is made that all deformations are very small in comparison with the dimensions of the members.

It is further assumed that:

(1) The moment of inertia of each member is constant along its length, although not necessarily the same for all members;

(2) The joints in the frame are either perfectly rigid, free to rotate, or belong to both categories; in other words, flexible joints are not considered. No such restriction is imposed upon the supports; and

(3) For the purpose of this analysis, the given frame may be replaced by an idealized system formed by the gravity axes of the members.

These assumptions are common to most methods for the analysis of frames. With some modifications, the method presented here may be extended to remove restrictions (1) and (2).

Compatibility Equations

The axial strains and unit deflections of members resulting from the displacements of the joints are subject to conditions of compatibility derived in the following manner.

Let AB and A′B′ in Fig. 2 represent any member in a frame, respectively, before and after displacement. From point A′, draw a line A′B″ parallel to AB and equal to it in length. Point B‴, the foot of a perpendicular from B′ on A′B″, defines the change in length e_1 of member AB and the transverse

movement Δ_1 of joint B. In Fig. 2, these elements are indicated in their positive directions, that is, elongation for e_1 and clockwise rotation for Δ_1.

Let δ_1 designate the unit deflection of the member, defined by $\delta_1 = \frac{\Delta_1}{l_1}$; ϵ_1 its axial strain, defined by $\epsilon_1 = \frac{e_1}{l_1}$; and let λ_1 be the inclination of AB on

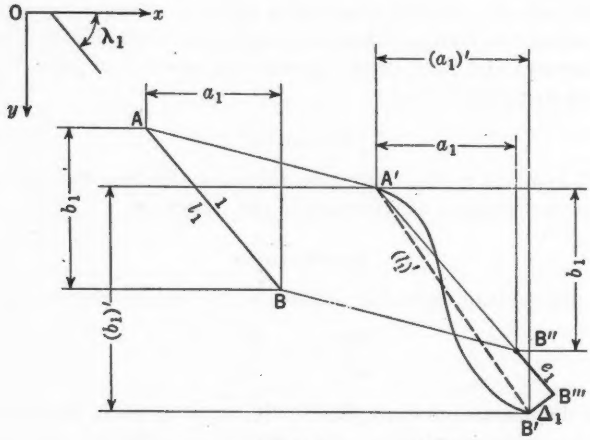


FIG. 2

the x-axis. The following relations may be written between the rectangular projections of AB and A'B':

$$(a_1)' = a_1 + e_1 \cos \lambda_1 - \Delta_1 \sin \lambda_1 \qquad (2a)$$

and

$$(b_1)' = b_1 + e_1 \sin \lambda_1 + \Delta_1 \cos \lambda_1 \qquad (2b)$$

or, in terms of δ_1 and ϵ_1:

$$(a_1)' - a_1 = a_1 \epsilon_1 - b_1 \delta_1 \qquad (3a)$$

and

$$(b_1)' - b_1 = b_1 \epsilon_1 + a_1 \delta_1 \qquad (3b)$$

Eqs. 2 and 3 apply to any member of the frame, regardless of its position or orientation, provided its rectangular projections are taken with the proper signs in relation to the reference axes. Apply these relations to a closed circuit of bars (given or imaginary); denote by $\sum_o$ a complete summation around this circuit, continuously in a clockwise direction. Since, for a closed circuit, $\sum_o a = 0$; $\sum_o a' = 0$; $\sum_o b = 0$; and $\sum_o b' = 0$, the following relations may be derived from Eqs. 3:

$$\textstyle\sum_o(a\,\epsilon) - \sum_o(b\,\delta) = 0 \qquad (4a)$$

and

$$\textstyle\sum_o(b\,\epsilon) + \sum_o(a\,\delta) = 0 \qquad (4b)$$

These relations simply mean that the axial and lateral strains of the bars are compatible in maintaining a closed circuit. It seems appropriate, therefore, to call Eqs. 4 "compatibility equations."

Consider a closed circuit, ABCDA, Fig. 3(*a*), in which the sides may be some of the given members in a frame or simply imaginary diagonals connecting two joints. Designate any two consecutive sides by the letters p and q. Eqs. 4 may be written in a simpler manner if the differences $\delta_p - \delta_q = \delta_{p-q}$ and $\epsilon_p - \epsilon_q = \epsilon_{p-q}$ are introduced instead of the elements δ and ϵ. At the joint

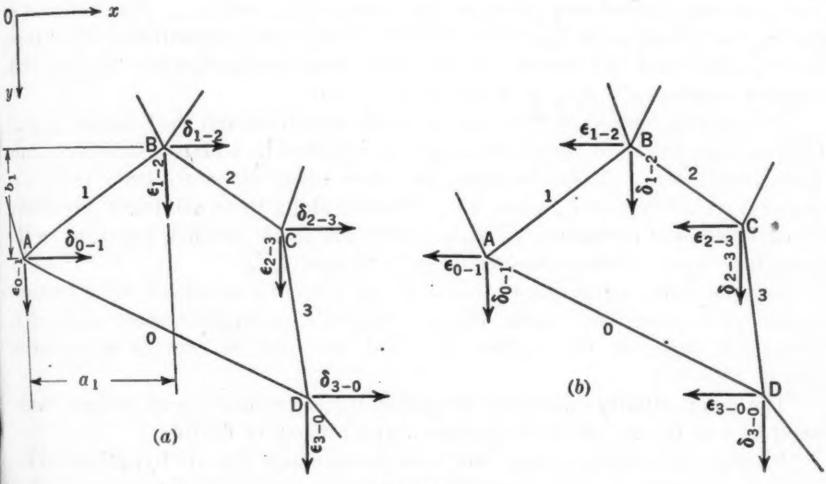


FIG. 3

where members p and q meet, apply the orthogonal vectors δ_{p-q} and ϵ_{p-q} in the direction of the arbitrary axes x and y. Treat all other joints in the same manner, as indicated in Fig. 3(*a*). It will be shown that the resultant moment of all these vectors about point O is zero if the compatibility equations are satisfied.

This moment is

$$M = x_a\,\epsilon_{0-1} + x_b\,\epsilon_{1-2} + x_c\,\epsilon_{2-3} + x_d\,\epsilon_{3-0} - (y_a\,\delta_{0-1} + y_b\,\delta_{1-2} + y_c\,\delta_{2-3} + y_d\,\delta_{3-0}) \quad (5)$$

in which x_a, y_a, etc., are the coordinates of joints A, etc.; and ϵ_{0-1}, δ_{0-1}, etc., are equal, respectively, to $\epsilon_0 - \epsilon_1$, $\delta_0 - \delta_1$, etc. Similarly, if $x_a - x_d$, etc., are expressed as x_{a-d}, etc., Eq. 5 may be transformed as follows:

$$M = x_{a-d}\,\epsilon_0 + x_{b-a}\,\epsilon_1 + x_{c-b}\,\epsilon_2 + x_{d-c}\,\epsilon_3 - (y_{a-d}\,\delta_0 + y_{b-a}\,\delta_1 + y_{c-b}\,\delta_2 + y_{d-c}\,\delta_3) \quad (6)$$

or again

$$M = \textstyle\sum_o(a\,\epsilon) - \sum_o(b\,\delta) \quad (7)$$

Eq. 7, which represents the moment about O of the vectors δ_{p-q} and ϵ_{p-q}, directed as in Fig. 3(*a*), is equal to zero by virtue of Eq. 4*a*.

If all these vectors are rotated bodily 90° clockwise, as in Fig. 3(*b*), it may be shown in a similar manner that their resultant moment about O reduces to

the expression

$$M' = \sum_o (b\,\epsilon) + \sum_o (a\,\delta) \quad \text{.................... (8)}$$

which is also equal to zero because of Eq. 4*b*.

It is thus seen that, if a certain number of the joints of a frame are considered as forming a closed circuit (by drawing imaginary diagonals, if necessary), the two moments (about any point) of the vectors δ_{p-q} and ϵ_{p-q}, directed first as in Fig. 3(*a*), then as in Fig. 3(*b*), reduce to zero if the compatibility conditions are satisfied; and vice versa. In addition, these vectors satisfy the two self-evident relations, $\sum_o \delta_{p-q} = 0$; and $\sum_o \epsilon_{p-q} = 0$.

The system formed by these two moment equations and the relation $\sum_o \delta_{p-q}$ is equivalent to a more symmetrical system obtained by equating three resultant moments to zero. These moments are taken about three arbitrary points, of the orthogonal vectors δ_{p-q} and ϵ_{p-q}, directed along three arbitrary directions. In each moment operation, all δ_{p-q}-vectors are made parallel, pointing in the same direction. This applies similarly to vectors ϵ_{p-q}.

Compatibility equations, expressed in the form of a system of three moment equations as described herein, can be written conveniently when applied to triangular elements, the vertices of which are used as centers of moments (see Example 1).

The compatibility equations developed in this section apply to any frame, regardless of the nature of the joints—rigid, hinged, or flexible.

Example 1.—After a frame has been decomposed into its hypothetical triangular elements, the compatibility conditions are applied to one of these

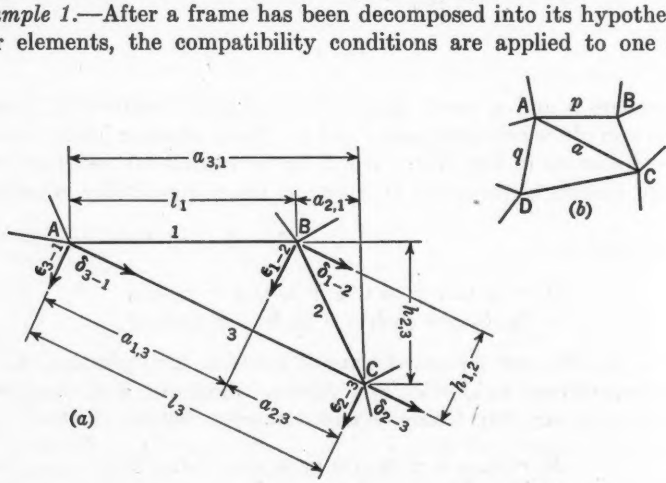


FIG. 4

elements. Fig. 4(*a*) represents a triangle ABC, formed by any three joints of a frame. The orthogonal vectors δ_{p-q} and ϵ_{p-q} are applied at the respective joints as shown. Equate their moment about point A to zero; thus:

$$M = h_{1,2}\,\delta_{1-2} + a_{1,3}\,\epsilon_{1-2} + l_3\,\epsilon_{2-3} = 0 \quad \text{.............. (9)}$$

or

$$\delta_{1-2} = \frac{l_3\,\epsilon_3 - a_{1,3}\,\epsilon_1 - a_{2,3}\,\epsilon_2}{h_{1,2}} \quad \text{................ (10a)}$$

If these vectors, fixed at right angles to each other, are rotated to bring the δ_{p-q}-vectors parallel with line BA, and their moment about point B is equated to zero:

$$\delta_{2-3} = \frac{l_1\,\epsilon_1 + a_{2,1}\,\epsilon_2 - a_{3,1}\,\epsilon_3}{h_{2,3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

A third expression of a similar type is obtained by equating to zero the moment about point C of the vectors δ_{p-q} and ϵ_{p-q}, rotated so that δ_{p-q} becomes parallel to line CB.

It may be concluded that δ_{p-q} at any angle formed by two given members in a frame meeting at a joint may be written in terms of the axial strains of the members forming the angle and that of the diagonal opposite. When the members are not contiguous, such as AD and AB in Fig. 4(*b*), δ_{p-q} may be written as $\delta_{p-a} + \delta_{a-q}$, in which each term may be expressed using the axial strains ϵ of the lines forming the triangles ABC and ACD.

If one (or several) of the sides of a triangular element happens to be a bar joining two supports, the δ-terms and ϵ-terms of this side will represent, respectively, the unit deflection and the axial strain of the line of supports. Both of these elements can be expressed easily as functions of the linear movements of the supports. In the development of this analysis, it will be found expedient to assume that the unit deflection and axial strain of bars (given or imaginary) joining two supports have been replaced in terms of the support movements. As a result, the compatibility equations will contain these movements explicitly, when applied to panels adjacent to support lines.

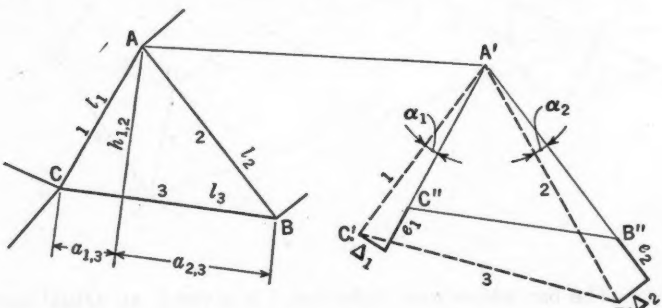


FIG. 5

A geometric representation of δ_{p-q} and ϵ_{p-q} may be drawn in terms of the deformation of a triangle. Let ABC, Fig. 5, be a triangular element in the frame; A′B′C′ is the triangle formed by joining the deflected positions of the vertices; and A′B″C″ the position of ABC translated until joint A coincides with joint A′. By definition,

$$\delta_{1-2}\ (= \delta_1 - \delta_2) = \frac{\Delta_1}{l_1} - \frac{\Delta_2}{l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

but $\frac{\Delta_1}{l_1}$ and $\frac{\Delta_2}{l_2}$ are respectively equal to angles α_1 and α_2. Therefore, $\delta_{1-2} =$

$\alpha_1 - \alpha_2$ = angle C′A′B′ − angle CAB = ΔA, in which ΔA is the variation of the original angle at joint A compared to that formed by straight lines connecting the new positions of the joints. Again, by application of compatibility equations, ϵ_{1-2} may be written easily as

$$\epsilon_{1-2} = \frac{a_{1,3}\,\delta_{1-3} + a_{2,3}\,\delta_{2-3}}{h_{1,2}} \qquad (12)$$

If δ_{1-3} and δ_{2-3} are referred to their appropriate angles C and B (Fig. 5) as $-\Delta C$ and $+\Delta B$, respectively, Eq. 12 can be expressed as:

$$\epsilon_{1-2} = \frac{\Delta B}{\tan B} - \frac{\Delta C}{\tan C} \qquad (13)$$

Example 2.—An important function of the compatibility conditions consists in determining the relative displacement of two arbitrary joints in a frame in terms of elements δ_{p-q} and axial strains ϵ, assuming that the unit deflection δ of one of the bars in the frame is given. For instance, referring to Fig. 6, let it be required to find the displacement of joint D with respect to joint A. Two cases are distinguished:

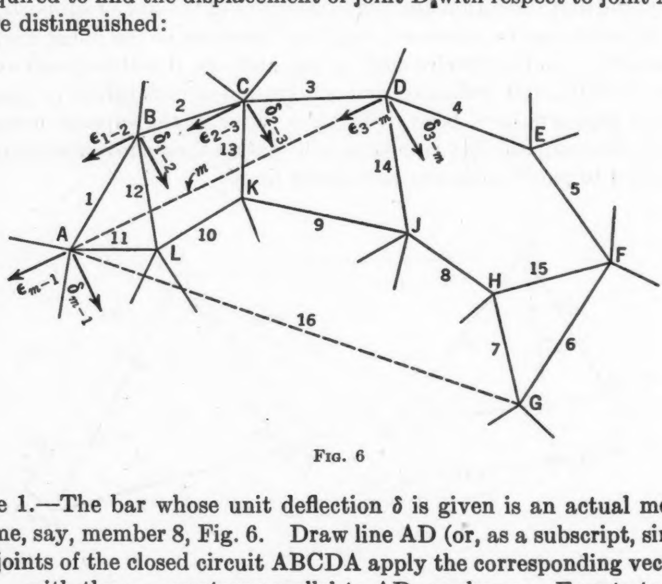


FIG. 6

Case 1.—The bar whose unit deflection δ is given is an actual member in the frame, say, member 8, Fig. 6. Draw line AD (or, as a subscript, simply m). At the joints of the closed circuit ABCDA apply the corresponding vectors δ_{p-q} and ϵ_{p-q}, with the ϵ_{p-q}-vectors parallel to AD, as shown. Equate to zero the moment of these vectors about joint A; ϵ_{3-m} will disappear and δ_{3-m} will be given as a linear function of the vectors applied between joints A and D, such as,

$$\delta_{3-m} \quad \text{or} \quad \delta_3 - \delta_m = F(\delta_{1-2}\ ,\ \epsilon_{1-2}\ ,\ \delta_{2-3}\ ,\ \epsilon_{2-3}) \qquad (14a)$$

In Eq. 14a, δ_3 may be expressed in terms of δ_8 (assumed given) and some δ_{p-q}-elements, as follows:

$$\delta_3 = \delta_{3-14} + \delta_{14-8} + \delta_8 \qquad (14b)$$

Substitute this value of δ_3 in Eq. 14a and obtain δ_m. The product $l_m\,\delta_m$ is

then the movement of joint D at right angles to line AD. Rotating the vectors, in Fig. 6, 90° clockwise and again taking their moment about joint A, δ_{3-m} will disappear and ϵ_{3-m} (or, $\epsilon_3 - \epsilon_m$) will be found as a linear expression of the vectors between joints A and D. The movement of joint D along line AD is then given by $l_m \epsilon_m$.

Case 2.—The bar whose unit deflection δ is given is an imaginary diagonal, say, line 16, Fig. 6, drawn between two joints. Eq. 14*a* holds also for this case; Eq. 14*b* will have to be replaced by:

$$\delta_3 = \delta_{3-14} + \delta_{14-8} + \delta_{8-7} + \delta_{7-16} + \delta_{16} \qquad (14c)$$

In Eq. 14*c*, δ_{16} is assumed known; all the terms in the second member, except δ_{7-16}, refer to actual angles in the frame, these being the only ones which will be necessary to consider in this analysis; δ_{7-16} can be found, and then eliminated in Eq. 14*c*, by locating the orthogonal vectors δ_{p-q} and ϵ_{p-q} at the joints of the closed circuit ALKJHGA (with the ϵ_{p-q}-vectors parallel to line AG) and equating to zero their moment about joint A. The movement of joint D along line AD is found as in case 1.

Corollary to Example 2.—Assuming that the given frame is completely decomposed into triangles by means of imaginary diagonals, the δ_{p-q}-elements at all angles in the frame may be written, as shown in Example 1, in terms of the axial strains of the bars (given or imaginary), and the linear movements of the supports. Therefore, as a corollary to Example 2, it may be concluded that the relative displacement of any two joints may be found in terms of the axial strains of the actual members, those of the imaginary diagonals, and the linear movements of the supports, knowing the unit deflection of any one bar in the frame (for this bar, one joining two supports may be used).

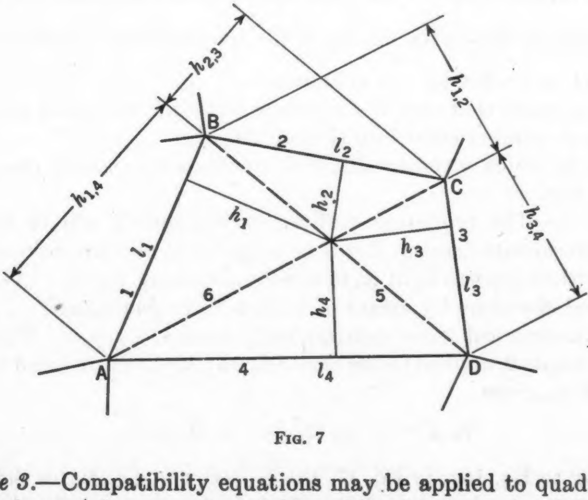


FIG. 7

Example 3.—Compatibility equations may be applied to quadrilateral elements, or polygons of any number of sides, either for the study of their deformation, or, more immediately, as a preliminary for the analysis of redundant polygonal panels. A quadrilateral element is shown in Fig. 7. Imagine it

divided into the four triangles ABC, BCD, CDA, DAB. Express δ_{1-2}, δ_{2-3}, δ_{3-4}, and δ_{4-1} by applying the method of Example 1 to each of these triangles, and equate their sum to zero; the following relation is obtained after simplification:

$$2\,A_e\left(\frac{\epsilon_5}{h_{1,4}\,h_{2,3}}+\frac{\epsilon_6}{h_{1,2}\,h_{3,4}}\right)=\frac{l_1\,\epsilon_1}{h_1}+\frac{l_2\,\epsilon_2}{h_2}+\frac{l_3\,\epsilon_3}{h_3}+\frac{l_4\,\epsilon_4}{h_4}\ldots\ldots(15)$$

In Eq. 15, A_e represents the area of the quadrilateral and ϵ_5 and ϵ_6 the axial strains of the diagonals. If the axial deformations of the sides are negligible, it takes the form

$$\frac{\epsilon_5}{h_{1,4}\,h_{2,3}}+\frac{\epsilon_6}{h_{1,2}\,h_{3,4}}=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

which correlates the axial strains of the diagonals in a quadrilateral with rigid sides.

Example 4.—The results of Example 3 will be used for an outline of the determination of the axial forces in the pin-connected truss with one redundant member, shown in Fig. 8. If the redundant member BD is replaced by a pair of opposite forces X, the symbol X being the unknown force in the member, the axial strains in each of the members 1, 2, 3, 4, and 5 will be of the form $\dfrac{T^1\,X+T^0}{E\,A}$; in this expression, A is the cross-sectional area of the member, and T^1 and T^0 are, respectively, the axial forces found, first, by assuming $X = 1$ and all external loads removed, and second, by assuming $X = 0$ and allowing the external loading to act. The axial strain in the redundant member itself is $\dfrac{X}{E\,A}$. Substitute these axial strains in the compatibility equation (Eq. 15) for panel ABCD, and solve for the unknown X.

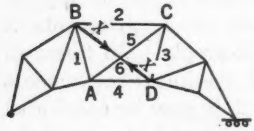


FIG. 8

It may be noted that only the members bounding the panel which contains the redundant member enter into the calculations.

The case in which several redundant members are present may be treated in a similar manner.

Example 5.—The procedure outlined in Example 2 will be applied to a statically determinate frame. Let it be required to find the vertical deflection of joint D with respect to joint A, in the Pratt truss of Fig. 9. (This truss has been analyzed elsewhere by means of Williot-Mohr diagrams.[3])

In this symmetrical truss, symmetrically loaded, $\delta_{11} = 0$. The procedure of case 1, Example 2, applied to the closed circuit ABCDA, reduced to a straight line in this case, gives

$$\delta_{8-m}=-\tfrac{1}{75}\,(25\,\delta_{2-4}+50\,\delta_{4-8})\ldots\ldots\ldots\ldots\ldots(17)$$

corresponding to Eq. 14a; in Eq. 17, the subscript m designates the imaginary line drawn between joints A and D. The elements ϵ_{p-q} of the circuit do not

[3] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 9th Ed., John Wiley & Sons, Inc., New York, N. Y., Pt. I, p. 306; also Pts. II and III.

appear because their moment about A is zero. Also

$$\delta_8 = \delta_{8-9} + \delta_{9-11} + \delta_{11} = \delta_{8-9} + \delta_{9-11} \quad \text{.............} (18)$$

corresponding to Eq. 14*b*. Combining Eqs. 17 and 18,

$$\delta_m = \delta_{8-9} + \delta_{9-11} + \tfrac{1}{3}\,\delta_{2-4} + \tfrac{2}{3}\,\delta_{4-8} \quad \text{.............} (19a)$$

or

$$\delta_m = \delta_{8-9} + \delta_{9-11} + \tfrac{1}{3}\,(\delta_{2-3} + \delta_{3-4}) + \tfrac{2}{3}\,(\delta_{4-5} + \delta_{5-7} + \delta_{7-8}) \quad \text{...} (19b)$$

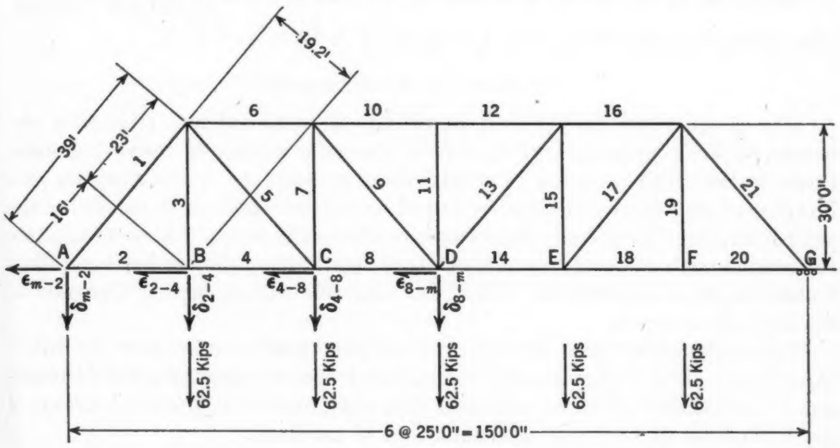


FIG. 9

All δ_{p-q}-terms in Eq. 19*b* are now expressed as functions of the axial strains of the members, using the method of Example 1 in the several triangles forming the truss; their expressions are:

$$\delta_{8-9} = -\frac{30\,\epsilon_7 - 30\,\epsilon_9}{25} \quad \text{.....................} (20a)$$

$$\delta_{9-11} = -\frac{25\,\epsilon_{10} - 25\,\epsilon_9}{30} \quad \text{.....................} (20b)$$

$$\delta_{2-3} = -\frac{39\,\epsilon_1 - 16\,\epsilon_2 - 23\,\epsilon_3}{19.2} \quad \text{..................} (20c)$$

$$\delta_{3-4} = -\frac{39\,\epsilon_5 - 23\,\epsilon_3 - 16\,\epsilon_4}{19.2} \quad \text{..............} (20d)$$

$$\delta_{4-5} = -\frac{30\,\epsilon_3 - 30\,\epsilon_5}{25} \quad \text{.....................} (20e)$$

$$\delta_{5-7} = -\frac{25\,\epsilon_6 - 25\,\epsilon_5}{30} \quad \text{.....................} (20f)$$

and

$$\delta_{7-8} = -\frac{39\,\epsilon_9 - 23\,\epsilon_7 - 16\,\epsilon_8}{19.2} \quad \text{..............} (20g)$$

The substitution of these values in Eq. 19*b* will give δ_m; thus:

$$\delta_m = -0.68\,\epsilon_1 + 0.28\,\epsilon_2 + 0.28\,\epsilon_4 + 0.68\,\epsilon_5 - 0.56\,\epsilon_6 - 0.40\,\epsilon_7 + 0.56\,\epsilon_8 + 0.68\,\epsilon_9 - 0.83\,\epsilon_{10} \qquad (21)$$

In this problem, the axial forces T in the members can be found immediately by statics; the corresponding axial strains are of the form $\epsilon = \frac{T}{EA}$.

Substituting the numerical values[3] for the ϵ-terms, $\delta_m = \frac{27{,}820}{29{,}000{,}000} = \frac{9.55}{10{,}000}$. The vertical movement of joint D is then $75 \times 12\,\delta_m = 0.86$ in.

Topology of Frameworks

The given frame having been completely decomposed into triangular elements by drawing imaginary diagonals, the axial strains of these diagonals, those of the given members, and the linear movements of the supports (see Example 1 for the reason of their introduction), are used as elements for the determination of the displacements and stresses in the frame. Their immediate connection with some of the important aspects of the problem has been illustrated in preceding sections. Their essential rôle will be further disclosed as the analysis proceeds.

These elements—axial strains and support movements—may be interdependent. This may be noted, for instance, by considering the plane deformation of a quadrilateral panel with two diagonals; evidently, the axial strains of any five of the six bars will determine that of the sixth.

The reduction of these elements to a minimum number which are independent, the determination of this number, the exposition of other properties connected with the configuration of the frame (such as relations between number of regions, members, joints, etc.) and leading ultimately to the solution of the independent elements—all result from a study of the topology of the frame.

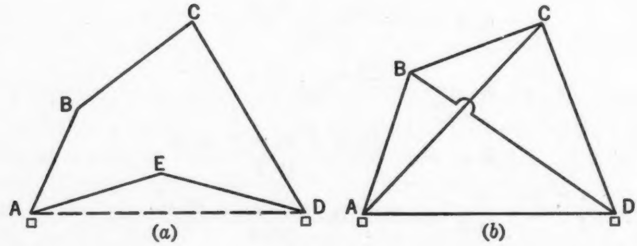


Fig. 10

A region in a framework is defined in this paper as the space within a closed circuit formed by some of the given members; or, within a closed circuit formed by some of the given members and an imaginary line joining two supports. For instance, the frame in Fig. 10(*a*) has three regions—ABCDEA, ABCDA, AEDA; that of Fig. 10(*b*) has five—namely, ABCA, ACDA, BCDB, ABDA, and ABCDA. A distinct region is defined as one whose boundary contains at least one bar (not joining 2 supports) which has not already been used in

circumscribing other regions; on this basis, the first frame has only two distinct regions, and the second only three, among the several regions enumerated for each of them. It is immaterial which particular regions are considered as distinct among all others resulting from the configuration of the frame; for reasons of simplicity, it is well to choose those with the smallest number of sides. In this analysis, only distinct regions will be considered.

Let:

n_r = total number of distinct regions in a frame, exclusive of those with all their vertices at supports;

n_m = total number of given members, exclusive of those connected to a support at each end;

n_p = total number of joints, exclusive of the supports.

(The reason for the exclusion from this list of members joining two supports, is that their axial strains and unit deflections are assumed to be expressed in terms of the linear movements of the supports. Similarly, the supports are not included among the number of the joints for the reason that the equilibrium and displacement requirements at the supports will be disposed of in the establishment of boundary conditions. The limitation imposed upon the number of regions is due to similar conditions.)

Proposition (*a*).—Between the numbers of regions, members and joints as defined above there exists the following relation:

$$n_r = n_m - n_p \qquad (22)$$

(Compare Eq. 22 to Euler's equation[2] relative to the number of faces, edges, and vertices of polyhedra.)

Eq. 22 is, of course, evident in the case of a rudimentary system composed of one member and one support ($n_r = 0$, $n_m = 1$, $n_p = 1$). It will be sufficient to show that it remains true when successive members, joints, and supports are added to it to form an arbitrary frame. For instance, at the free end of this system attach a closed region of n members. Then $n_r = 1$, $n_m = n + 1$, $n_p = n$, and Eq. 22 is satisfied. Next, add one member between two of the joints, or between one of the joints and the support, or between one of the joints and a new support: There are now one more region and one more member; the number of the joints has not changed; and the relation still applies. Add a member between two supports: The number n_r does not change; neither does n_m, nor n_p; and the relation is again valid. It will remain valid after the addition of a new member connecting a joint or a support with some point on one of the members, thus creating a new joint at that point and converting one member into two (this will result in one more region, two additional members, and one new joint).

Similarly, it may be shown that Eq. 22 remains valid at each stage of growth resulting from any other type of addition to the frame.

For example, in Fig. 9, $n_m = 21$, $n_p = 10$, $n_r = 21 - 10 = 11$, these regions being the ten triangles subdividing the truss, and the space between the bottom chord and the line of supports (which coincides in this case with the bottom chord).

Proposition (*b*).—From Eq. 22, the total number of distinct triangular elements n_t (excluding those with all their vertices at supports) into which the

frame is decomposed by diagonals is given by:

$$n_t = (n_m + n_{mt}) - n_p \quad \text{.....................} (23)$$

In this relation n_{mt} represents the total number of imaginary diagonals required for the complete decomposition of the frame into triangles, exclusive of those joining two supports.

After the frame has been divided into triangles, a total of $2\,n_t$, compatibility equations (Eqs. 4) may be written for the n_t triangular elements forming the transformed frame. These equations will be in terms of the unit deflections δ and δ_t, and the axial strains ϵ and ϵ_t of the given members and the imaginary diagonals, respectively. They will also involve the linear movements of the supports. Consider the $(n_m + n_{mt})$-elements δ and δ_t eliminated between these $2\,(n_m + n_{mt} - n_p)$ equations; and there will remain $2\,(n_m + n_{mt} - n_p) - (n_m + n_{mt}) = n_m + n_{mt} - 2\,n_p$ linear relations between the elements ϵ and ϵ_t and the linear movements of the supports; call n_i the number: $n_m + n_{mt} - 2n_p$.

Proposition (*c*).—As a result of Proposition (*b*), calling n_σ the number of the support movements, it may be concluded that a number n_i of the $n_m + n_{mt}$ axial strains ϵ and ϵ_t are dependent upon, and may be determined as functions of, the remaining $n_m + n_{mt} + n_\sigma - n_i = n_m + n_{mt} + n_\sigma - (n_m + n_{mt} - 2\,n_p) = 2\,n_p + n_\sigma$ elements (axial strains and linear movements of supports), considered as independent. This last number reduces to $(2\,n_p - n_m) + n_\sigma$ in the assumption of negligible axial strains for the n_m given members. In other words, the degree of freedom of a frame in joint displacement is $2\,n_p + n_\sigma$ in the general case, and $(2\,n_p - n_m) + n_\sigma$ if the axial deformations of the n_m given members are assumed to be negligible; (this is, of course, in addition to its displacement as a whole).

In most cases, these n_i-relations may be obtained easily by expressing the value of δ_{p-q} for each angle at an interior joint or interior support, in terms of the ϵ-elements and ϵ_t-elements of the neighboring bars and the linear movements of the supports (see Example 1), and writing the self-evident relation $\sum \delta_{p-q} = 0$ at these points.

An illustration is afforded by the frame in Fig. 13 (given subsequently), in which $n_m = 5, n_p = 3, n_t = 4, n_{mt} = 2$; number of regions $n_r = n_m - n_p = 2$; number of triangles $n_t = n_m + n_{mt} - n_p = 4$; $n_i = n_m + n_{mt} - 2\,n_p = 1$ ($\sum \delta_{p-q} = 0$ at support F, Fig. 13); degree of freedom in joint displacement, $2\,n_p = 6$, or $2\,n_p - n_m = 1$.

The application of this analysis to articulated frames leads to their classification as statically determinate, redundant, unstable, and redundant-unstable, and to criteria for such classification.[4]

Boundary Conditions

In preparing to find end moments in terms of the unknowns adopted in this analysis, boundary conditions will be expressed in a form excluding the reac-

[4] "Displacements and Stresses in the General Two-Dimensional Framework," by Yves Nubar; submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy, Faculty of Pure Science, Columbia Univ., New York, N. Y., 1941. On file for reference in Engineering Societies Library New York, N. Y.

tions of the supports; their expression will involve only axial strains, linear and angular movements of supports, and end moments.

A support of a general type is shown at point a in Fig. 11. Let X_a, Y_a, and M_a be the reactions at the support and e_{xa}, e_{ya}, ω_a the corresponding linear

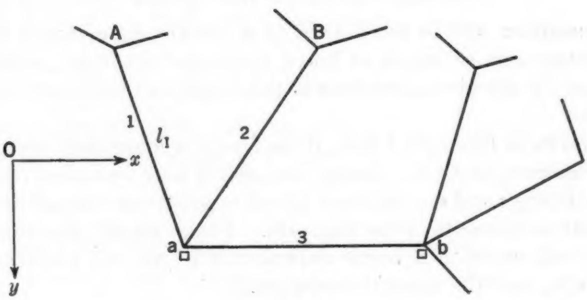


FIG. 11

and angular movements. At this support, boundary conditions in their general form may be represented by three expressions between these six elements, such as

$$\left.\begin{array}{l} F_1(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) = 0 \\ F_2(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) = 0 \\ F_3(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) = 0 \end{array}\right\} \dots\dots\dots\dots\dots (24)$$

In practical cases, these expressions are of much simpler type; for instance, at a hinged, immovable joint, they reduce to: $e_{xa} = 0$, $e_{ya} = 0$, $M_a = 0$; and, at a support on rollers, they may be written as: $e_{ya} = 0$, $X_a = 0$, $M_a = 0$.

Reactions X_a, Y_a, and M_a can be eliminated from Eqs. 24 as follows: The moments and forces in each of the members (for instance, member 1) meeting at the support are of the form: Axial force, $E\,A_1\,\epsilon_1$; end moments, M_{1a} and M_{1A}; transverse end force, $\frac{M_{1a} + M_{1A}}{l_1}$ (this expression will also involve lateral loads on the member, if any). Using these expressions, the equilibrium conditions at the support are:

$$X_a + \sum(E\,A_1\,\epsilon_1)_x + \sum\left(\frac{M_{1a} + M_{1A}}{l_1}\right)_x = 0 \dots\dots\dots (25a)$$

$$Y_a + \sum(E\,A_1\,\epsilon_1)_y + \sum\left(\frac{M_{1a} + M_{1A}}{l_1}\right)_y = 0 \dots\dots\dots (25b)$$

and

$$M_a + \sum M_{1a} = 0 \dots\dots\dots\dots\dots\dots\dots\dots (25c)$$

Solve for X_a, Y_a, and M_a and introduce the result in Eqs. 24. These relations will represent the boundary conditions at support a, Fig. 11, but will involve only the linear and angular movements of the support, the axial strains and end moments in the members connected there.

If line 3, Fig. 11, is only an imaginary diagonal joining two supports, the forces and moments corresponding to it will naturally disappear from Eqs.

24 and 25. If it is an actual member, its end moments, transverse and axial forces may be assumed to have been written in terms of the angular and linear movements of the supports, using the slope deflection[5] equations of the member.

Determination of End Moments

The proposition will be established that the end moments in the members of a rigid frame can be found as linear functions of the δ_{p-q}-elements at the angles formed by the given members of the frame and the linear movements of the supports.

It was shown in Example 1 that, if the frame is completely decomposed into triangular elements, these δ_{p-q}-terms themselves may be expressed as functions of the axial strains ϵ and ϵ_t of the bars (given members and imaginary diagonals) and the linear movements of the supports. Consequently, the aforementioned proposition will result in a linear dependence of all end moments upon the unknowns, ϵ, ϵ_t, and the support movements.

When such a dependence is established, it will follow that the axial strains ϵ and ϵ_t, together with the linear movements of the supports, can specify completely the system of displacements and stresses in the general two-dimensional frame; this conclusion is valid, because:

(1) The axial force T in a member equals $E\,A\,\epsilon$;

(2) The transverse forces at the ends of a member may be found from the end moments by means of the equilibrium conditions of the member;

(3) The relative displacement of any two joints can be determined in terms of the axial strains ϵ and ϵ_t and the linear movements of the supports (see "Corollary to Example 2");

(4) The relative rotation of two joints may be found from the end moments of the members by application of slope deflection[5] equations (see, for instance, Eqs. 28, given subsequently);

(5) The boundary conditions, as written in the preceding section, furnish the angular and linear movements of the supports in terms of axial strains, and end moments.

The aforementioned proposition is demonstrated by first establishing the following property of rigid joints:

I.—If, at a joint of n members, there exists one linear relation between the end moments of each of $n-1$ members (a total of $n-1$ relations), another linear relation may be found between the end moments of the nth member. This expression involves the coefficients of the $n-1$ linear relations and the terms δ_{p-q} at the joint.

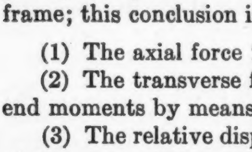


Fig. 12

A joint of three members will be considered for simplicity, the demonstration being similar for a joint of any number of members. Let M_{1A} and M_{1C} designate the moments exerted by member 1, Fig. 12, upon joints A and C, respectively; m_{1A} and m_{1C} designate the same moments if joints A and C do not rotate, and if member AC does not deflect laterally (fixed end moments);

[5] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1918.

ω (with the subscript of a joint) represents the rotation of the joint common to all members connected to it. These moments and rotations are taken as positive in a clockwise direction.

For members 1 and 2, Fig. 12, assume given two linear relations:

$$M_{1A} + C_{1C}\, M_{1C} = D_{1C} \qquad (26a)$$

and

$$M_{2B} + C_{2C}\, M_{2C} = D_{2C} \qquad (26b)$$

C_{1C}, D_{1C}, C_{2C}, D_{2C} representing given coefficients.

It is proposed to derive a similar relation

$$M_{3C} + C_{3D}\, M_{3D} = D_{3D} \qquad (26c)$$

for member 3—that is, to determine coefficients C_{3D} and D_{3D}.

With the adopted sign conventions, the slope deflection equations[5] for member 1 are

$$M_{1A} = -2\,E\,K_1\,(2\,\omega_A + \omega_C - 3\,\delta_1) + m_{1A} \qquad (27a)$$

and

$$M_{1C} = -2\,E\,K_1\,(2\,\omega_C + \omega_A - 3\,\delta_1) + m_{1C} \qquad (27b)$$

in which K_1 is the stiffness $\frac{I_1}{l_1}$ of member 1. Solving for ω_A and ω_C:

$$\omega_A = \delta_1 + \frac{[(M_{1C} - m_{1C}) - 2\,(M_{1A} - m_{1A})]}{6\,E\,K_1} \qquad (28a)$$

and

$$\omega_C = \delta_1 + \frac{[(M_{1A} - m_{1A}) - 2\,(M_{1C} - m_{1C})]}{6\,E\,K_1} \qquad (28b)$$

Consideration of members 2 and 3 will furnish the following additional expressions for ω_C:

$$\omega_C = \delta_2 + \frac{[(M_{2B} - m_{2B}) - 2\,(M_{2C} - m_{2C})]}{6\,E\,K_2} \qquad (29a)$$

and

$$\omega_C = \delta_3 + \frac{[(M_{3D} - m_{3D}) - 2\,(M_{3C} - m_{3C})]}{6\,E\,K_3} \qquad (29b)$$

Subtract Eq. 29*b* from Eq. 28*b* and Eq. 29*b* from Eq. 29*a*:

$$6\,E\,\delta_{1-3} + \frac{1}{K_1}\,(M_{1A} - 2\,M_{1C}) + \frac{1}{K_3}\,(2\,M_{3C} - M_{3D}) = \frac{1}{K_1}\,(m_{1A} - 2\,m_{1C}) + \frac{1}{K_3}\,(2\,m_{3C} - m_{3D}) \qquad (30a)$$

and

$$6\,E\,\delta_{2-3} + \frac{1}{K_2}\,(M_{1B} - 2\,M_{2C}) + \frac{1}{K_3}\,(2\,M_{3C} - M_{3D}) = \frac{1}{K_2}\,(m_{2B} - 2\,m_{2C}) + \frac{1}{K_3}\,(2\,m_{3C} - m_{3D}) \qquad (30b)$$

By Eqs. 26*a* and 26*b*:

$$\frac{C_{1C}+2}{K_1} M_{1C} = \frac{1}{K_3}(2\,M_{3C} - M_{3D}) + 6\,E\,\delta_{1-3} + \frac{D_{1C}}{K_1} - \frac{1}{K_1}(m_{1A} - 2\,m_{1C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D}) \qquad (31a)$$

and

$$\frac{C_{2C}+2}{K_2} M_{2C} = \frac{1}{K_3}(2\,M_{3C} - M_{3D}) + 6\,E\,\delta_{2-3} + \frac{D_{2C}}{K_2} - \frac{1}{K_2}(m_{2B} - 2\,m_{3C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D}) \qquad (31b)$$

If M_{1C} and M_{2C} from Eqs. 31 are substituted in the equilibrium equation of the joint—

$$M_{1C} + M_{2C} + M_{3C} + M_C = 0 \qquad (32)$$

in which M_C represents, for generality, an external couple applied at joint C, the following is the result:

$$2\,M_{3C}\left(\frac{K_1}{C_{1C}+2} + \frac{K_2}{C_{2C}+2} + \frac{K_3}{2}\right) - M_{3D}\left(\frac{K_1}{C_{1C}+2} + \frac{K_2}{C_{2C}+2}\right) = -\,K_3\,U \qquad (33)$$

in which

$$U = +\,M_C + \frac{K_1}{C_{1C}+2}\left[6\,E\,\delta_{1-3} + \frac{1}{K_1}(D_{1C} - m_{1A} + 2\,m_{1C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D})\right] + \frac{K_2}{C_{2C}+2}\left[6\,E\,\delta_{2-3} + \frac{1}{K_2}(D_{2C} - m_{2B} + 2\,m_{2C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D})\right] \qquad (34)$$

Coefficients C_{3D} and D_{3D} in Eq. 26*c* are found by comparing this equation to Eq. 33; thus:

$$C_{3D} = -\,\frac{\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2}}{2\left(\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2} + \dfrac{K_3}{2}\right)} \qquad (35a)$$

and

$$D_{3D} = -\,\frac{K_3\,U}{2\left(\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2} + \dfrac{K_3}{2}\right)} \qquad (35b)$$

This establishes the property stated in paragraph I.

The relations developed in this section may be considered to represent compatibility conditions in joint rotations, since they imply members rigidly connected to each other.

In case another member CE (numbered 4) is hinged at joint C of Fig. 12 (this member not shown), it is to be considered as nonexistent, so far as the operations of this section are concerned; its coefficients C_{4E} and D_{4E} are evidently zero.

The generalization of Eqs. 35 for application to joints of any number of members is obvious by inspection of the structure of these formulas. Coefficients C depend only upon the stiffness of the members. Therefore, they may be written once and used for all cases of loading and lateral deflections in the frame.

Eqs. 35 represent recurrence relations which make it possible to correlate end moments in a series of members by passage from one group of members to adjacent ones. The process of determining the successive coefficients C and D for consecutive members may be described as follows:

The determination of coefficients C and D may start at a member connected to a support. Two cases will be considered, depending on the type of this support, taken as the origin.

Case 1.—In the first case, the origin is a simply connected support; that is, only one member is connected to it. If this member is hinged at the origin, its coefficients C and D equal zero; if this is not the case, to find its coefficients C and D, consideration is given to Eq. 28*a* relative to this member. In this equation, the support rotation ω (if not zero) can be replaced by the end moments and the linear movements of the support, using the appropriate boundary condition therein which correlates these elements. The lateral deflection δ in Eq. 28*a* can also be written in terms of some adjacent δ_{p-q}-elements and some support movements (this is true for even more complex types of supports, such as support a of Fig. 11, in which $\delta_1 = \delta_{1-2} + \delta_{2-3} + \delta_3$). The resulting expression will be a linear relation involving the end moments of the member; its coefficients are the required values of C and D for this member; they will be in terms of the δ_{p-q}-elements and the support movements.

Case 2.—In the second case, the origin is a multiply connected support; that is, it is situated on the periphery of one or more closed circuits, such as joint a of Fig. 11. The rotation of the support ω (if not zero), common to all members meeting it, is maintained in equations such as Eq. 28*a* corresponding to each one of these members. Their lateral deflections δ can be replaced, the same as in case 1, using neighboring δ_{p-q}-elements and linear movements of adjacent supports. The resulting linear equations, involving the end moments, will again furnish the coefficients C and D for each member, except that, in this case, C and D will also be functions of the support rotation ω as a parameter, used for "opening up" the closed circuits at the origin. It will be eliminated in the remaining operations which will be described in the next paragraph. In case all members are hinged at the common origin, their coefficients C and D naturally vanish.

Coefficients C and D for one or several starting members having been found, operations may proceed continuously in one direction, using Eqs. 35. Successive sets of C and D coefficients for each span are written as functions of those preceding, all of them in linear terms of δ_{p-q}-elements. Each set signifies one linear relation between the end moments of a member. A span will be reached

finally where the end moments are already correlated by another equation resulting from a given boundary condition. This will mean two equations in terms of two end moments. Their solution will yield these moments which then can be carried backward toward the origin, for the determination of all end moments in preceding spans. For case 2, all these moments will contain the rotation ω of the origin as a parameter. This rotation can now be determined by writing $\sum M = 0$ at the origin and then replacing it in the expressions of the moments.

The operations are similar to the foregoing when both the origin and terminal support are multiply connected, or when the frame contains closed regions. In such cases, a joint or support rotation can again be maintained in the formulas for the purpose of "opening up" closed regions; later it can be eliminated. Several adjacent closed regions may be "opened up" by the use as a parameter of a single joint rotation.

The general approach presented herein may be greatly simplified in each individual case of practical application.

The principal conclusion to be drawn from this discussion is that the end moments in a frame can be found as linear functions of the δ_{p-q}-elements at the angles and the linear movements of the supports. This proves the proposition stated at the head of this section.

A geometric interpretation of the property of rigid joints stated in paragraph I is the following: If, at a joint of n members, the moment lines of all but one of the members pass through known fixed points ($n - 1$ in all), the moment line of the remaining member will also pass through a fixed point which may be located using the coordinates of the known points, and the δ_{p-q}-elements at the joint. Upon this interpretation, a generalized theory of fixed points may be based for the determination of end moments by means of semi-graphical operations.[4] (See also the work[6] of L. H. Nishkian, and D. B. Steinman, Members, Am. Soc. C. E.)

Determination of Axial Strains and the Linear Movements of Supports

The essential function of the group of the axial strains ϵ and ϵ_t of the given and imaginary members and the linear movements of the supports, in specifying the system of displacements and stresses in a frame, has been described in the preceding part of this paper. The determination of this group of elements is the object of this section.

In some problems, the displacements of the joints and the supports are supplied as part of the data; in such cases, the analysis of the frame is comparatively simple: All δ_{p-q}-elements appearing in Eqs. 34 and 35 assume known values; and all end moments, axial forces, and shears are found at the conclusion of the operations of the preceding section. In other problems, all joints are hinged, and the axial forces in the members may be determined by simple statics; the axial strains are then available and can be used for the

[6] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1–206.

determination of all deflections (see "Corollary to Example 2" and "Example 5"). In general, however, the frame is rigid and the joints and supports are subject to unknown displacements. The determination of axial strains ϵ and ϵ_t and support movements is then necessary, so far as the present analysis is concerned.

At this stage of the analysis, end moments, hence transverse end forces in all given members, are assumed to have been written as functions of $n_m + n_{mt} + n_\tau$, axial strains ϵ and ϵ_t, and support movements. More precisely, they may be assumed to be expressed in terms of the quantity $2\,n_p + n_\sigma$ of these elements considered as independent (see "Topology of Frameworks: Proposition (c)").

In the given frame decomposed completely into triangular elements by means of m_{mt} imaginary diagonals, the axial strains in n_i of the bars are considered to be dependent upon the others (see Proposition (c)). Remove from the resulting figure n_i bars judiciously chosen, and replace all joints and supports by perfect hinges. The structure thus obtained will be called a conjugate frame (see Fig. 13, given subsequently). This frame has the same n_p joints as the original; but the number of its bars is $(n_m + n_{mt}) - n_i = n_m + n_{mt} - (n_m + n_{mt} - 2\,n_p) = 2\,n_p$. At the joints of the conjugate frame apply the corresponding joint loads of the original, and, in addition, load them with the transverse end force exerted by each given member. So far as the forces in the bars are concerned, the conjugate frame is statically determinate since it has $2\,n_p$ bars and n_p joints, each furnishing two equilibrium equations, $\sum X = 0$ and $\sum Y = 0$. The solution of this frame will yield $2\,n_p$ axial forces as functions of the $2\,n_p + n_\sigma$ independent unknowns. Equate to zero the axial forces corresponding to the imaginary diagonals; equate to $E\,A\,\epsilon$ of the corresponding member, those belonging to the given members; the result is a system of $2\,n_p$ equations for the determination of $2\,n_p$ unknown axial strains in terms of n_σ support movements. These movements can then be found by the use of the corresponding boundary conditions.

In case the axial strains of the given members are assumed to be negligible —a very frequent case in rigid frames—the number of independent axial strains decreases from $2\,n_p$ to $2\,n_p - n_m$. It will then be sufficient to solve only for the axial forces in the $n_{mt} - n_i\ (= 2\,n_p - n_m)$ imaginary diagonals and to equate them to zero. Often, a convenient way of doing this is to apply the equilibrium equation $\sum(M) = 0$ to a portion of the conjugate frame on one side of some line X–X, treated as a free body; if this line is drawn to intersect one of the imaginary diagonals, and if the center of moments is chosen with care, this equation may contain none (or only a few) of the axial forces in the members (these forces are of no immediate interest in this case). The result will be a relation involving only the loads at the joints of the conjugate frame, that is, a relation between the $(2\,n_p - n_m) + n_\sigma$ independent unknowns. By varying the line X–X and the center of moments, additional relations of the same type may be obtained (see Illustrative Problem, and Fig. 13, given subsequently).

The treatment of the case (mentioned at the beginning of this section), in which the joints of a frame are subjected to given displacements rather than loads, can now be amplified further. These displacements immediately specify all elements δ, δ_{p-q}, ϵ, ϵ_t. If the movements of the supports are also assumed

given, all end moments, axial and transverse end forces throughout the frame will be known in terms of these displacements; the $2 n_p$ equations of equilibrium at the joints of the conjugate frame will then furnish the $2 n_p$ components X and Y of the n_p joint constraints corresponding to, and resulting in, the joint displacements impressed upon the frame.

Application to the General Case

The method presented in this paper will be applied to the general case of a frame in which axial deformations are not negligible; the actual solution is best described by decomposing it into the following sequence. This will help to recapitulate the procedure of the analysis.

Step 1.—Divide the frame completely into simple triangular panels by means of imaginary diagonals. For the $2 n_p + n_\sigma$ independent unknowns (see "Topology of Frameworks: Proposition (*c*)"), choose (in addition to the n_σ linear movements of supports) the axial strains of $2 n_p$ of the given members, if $2 n_p \leqq n_m$. Choose the axial strains of all n_m given members and $2 n_p - n_m$ of the imaginary diagonals, if $2 n_p > n_m$. Some judgment must be exercised in this choice, with a view to convenience in the operations of step 2.

Step 2.—Proceeding as in Example 1, express all δ_{p-q}-elements, one at each angle formed by given members, in terms of the independent unknowns. This is done conveniently by first writing these δ_{p-q}-values using the axial strains of all the bars in the vicinity, and then eliminating the dependent axial strains by means of the n_i-relations (see "Topology of Frame-work: Proposition (*c*)").

Step 3.—By application of the procedure described under the heading, "Determination of End Moments," express all end moments as functions of δ_{p-q}-elements (hence, as functions of the independent unknowns); if the end moments determined thus involve also the rotations at some of the multiply connected supports (or at joints on the periphery of closed regions), these rotations may be found and eliminated by the use of $\sum M = 0$ at such supports or joints.

Step 4.—Using these end moments, express all transverse end forces as functions of the independent unknowns; load the conjugate frame and solve for the $2 n_p$ independent axial strains in terms of the n_σ linear movements of the supports (see heading, "Determination of Axial Strains and the Linear Movements of Supports").

Step 5.—Use the boundary conditions and determine the displacements of the supports. Of three boundary conditions at each support, one, relating to the support rotation and equivalent to $\sum M = 0$, will yield the rotation of the support, if not already found in step 3 of this sequence. The remaining two boundary conditions will furnish the linear movements of the support, and Eqs. 25 will give the reactions.

All stresses and displacements throughout the frame may now be found using the $2 n_p + n_\sigma$ independent unknowns just determined.

The general solution outlined herein can be simplified appreciably if one chooses to analyze a rigid frame, as a preliminary step, under the assumption of negligible axial deformations in the members. Such an analysis (outlined under a subsequent heading, "Application to a Less General Case") will yield

axial forces in the given members which, in general, are correct enough for a first approximation. The axial strains of these members may then be computed and used in the compatibility equations of step 2 (this sequence), which will then contain a small constant term (corresponding to these axial strains), in addition to the axial strains ϵ_t of imaginary diagonals and the movements of the supports. The solution can then proceed, yielding another set of axial forces, which may be used for a second cycle of the same operations, each cycle altering the small constant term in the compatibility equations to a still smaller extent. The procedure may be repeated as many times as desired. In general, one cycle is ample when dealing with most rigid frames.[4]

Application to a Less General Case

The analysis of a rigid frame in which the axial deformations of the n_m given members are assumed to be negligible proceeds along steps similar to those involved in the general case. The number of independent unknowns becomes $(2\,n_p - n_m) + n_\sigma$, comprising $(2\,n_p - n_m)$ axial strains (this is also the number of imaginary diagonals in the conjugate frame) and n_σ support movements; $2\,n_p$ must be greater than n_m, otherwise, the axial deformations of the given members may not all be assumed negligible. The determination of the independent unknowns is much simpler than in the general case, since, as has been noted before, it will be sufficient to write only that the axial forces in the imaginary diagonals vanish.

If they are of interest, the axial forces in the given members may be found by the solution of the conjugate frame, in the same manner as in any statically determinate frame.

Illustrative Problem

Although somewhat different from the ordinary type of structures, the following case was chosen in order to illustrate as many points of the preceding analysis as possible.

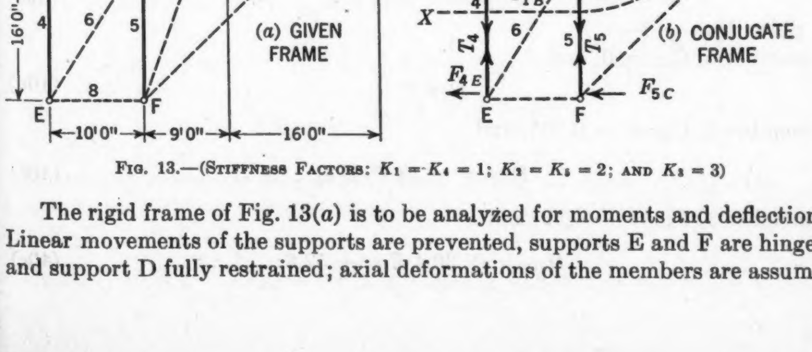


Fig. 13.—(Stiffness Factors: $K_1 = K_4 = 1$; $K_2 = K_5 = 2$; and $K_3 = 3$)

The rigid frame of Fig. 13(a) is to be analyzed for moments and deflections. Linear movements of the supports are prevented, supports E and F are hinged, and support D fully restrained; axial deformations of the members are assumed

to be negligible; units are in feet and kips. The steps in the procedure correspond to those in "Application to the General Case."

Step 1.—Draw imaginary diagonals 6, 7, 8, and 9. Due to the nature of the supports, $\delta_8 = \delta_9 = 0$, $\epsilon_8 = \epsilon_9 = 0$; and, because of negligible axial deformations, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = 0$. The number of independent unknowns is $2\,n_p - n_m + n_\sigma = 2 \times 3 - 5 + 0 = 1$. The axial strain ϵ_6 of imaginary diagonal 6 will be taken as this unknown.

Step 2.—By application of Example 1, the expressions for the several δ_{p-q}-elements (using scaled dimensions) are as follows:

$$\left.\begin{aligned} \delta_{8-4} &= \delta_{8-6} + \delta_{6-4} = -\,2.22\,\epsilon_6 \\ \delta_{4-1} &= \phantom{\delta_{8-6} + \delta_{6-4}} +\,2.22\,\epsilon_6 \\ \delta_{1-5} &= \delta_{1-6} + \delta_{6-5} = -\,2.22\,\epsilon_6 \\ -\,\delta_{8-5} = \delta_{5-8} &= \phantom{\delta_{8-6} + \delta_{6-4}} +\,2.22\,\epsilon_6 \end{aligned}\right\}\quad\text{(36)}$$

$$\left.\begin{aligned} -\,\delta_{5-9} = \delta_{9-5} &= \delta_{9-7} + \delta_{7-5} = -\,1.78\,\epsilon_7 \\ \delta_{5-2} &= \phantom{\delta_{9-7} + \delta_{7-5}} +\,4.92\,\epsilon_7 \\ \delta_{2-3} &= \delta_{2-7} + \delta_{7-3} = -\,4.92\,\epsilon_7 \\ \delta_{3-9} &= \phantom{\delta_{9-7} + \delta_{7-5}} +\,1.78\,\epsilon_7 \end{aligned}\right\}\quad\text{(37)}$$

The number of n_i-relations is $5 + 2 - 2 \times 3 = 1$. This is the equation $\sum \delta_{p-q} = 0$ at the interior support F, or $\delta_{8-5} + \delta_{5-9} = 0$; that is, $-\,2.22\,\epsilon_6 + 1.78\,\epsilon_7 = 0$. Hence: $\epsilon_7 = 1.245\,\epsilon_6$. Substituting this value of ϵ_7 in Eqs. 37:

$$\left.\begin{aligned} \delta_{9-5} &= -\,2.22\,\epsilon_6; \qquad & \delta_{5-2} &= +\,6.12\,\epsilon_6 \\ \delta_{2-3} &= -\,6.12\,\epsilon_6; \qquad & \delta_{3-9} &= +\,2.22\,\epsilon_6 \end{aligned}\right\}\quad\text{(38a)}$$

and

$$\delta_{1-2} = \delta_{1-5} + \delta_{5-2} = 3.90\,\epsilon_6 \quad\text{(38b)}$$

Step 3.—The coefficients C and D in each successive member are determined by Eqs. 34 and 35, in which all δ_{p-q}-elements are replaced by their equivalents, Eqs. 36 and 38. Operations begin at joint E and proceed toward joint D $\left(\text{note that } m_{3C} = +\frac{16 \times 40}{8} = +\,80,\ m_{3D} = -\,80\right)$:

Since $M_{4E} = 0$—
in member 4, $C_{4A} = 0$; and

$$D_{4A} = 0 \quad\text{(39a)}$$

in member 1, $C_{1B} = -\,0.25$; and

$$D_{1B} = -\,3.33\,E\,\epsilon_6 \quad\text{(39b)}$$

Since $M_{5F} = 0$—
in member 5, $C_{5B} = 0$; and

$$D_{5B} = 0 \quad\text{(40a)}$$

in member 2, $C_{2C} = -\,0.305$; and

$$D_{2C} = -\,18.75\,E\,\epsilon_6 \quad\text{(40b)}$$

in member 3, $C_{3D} = -\,0.22$; and

$$D_{3D} = +\,30.4\,E\,\epsilon_6 + 52.8 \quad\text{(40c)}$$

Using the values for C_{3D} and D_{3D} from Eqs. 40*c* in the equation $M_{3C} + C_{3D} \times M_{3D} = D_{3D}$, for member 3:

$$M_{3C} - 0.22\,M_{3D} = +\,52.8 + 30.4\,E\,\epsilon_6 \qquad (41a)$$

Making $\omega_D = 0$ in the slope deflection equation for member 3 (corresponding to Eq. 28*b*), a second relation is obtained between M_{3C} and M_{3D}:

$$M_{3C} - 2\,M_{3D} = +\,240 - 39.96\,E\,\epsilon_6 \qquad (41b)$$

Solving Eqs. 41:

$$M_{3C} = +\,29.7 + 39.15\,E\,\epsilon_6 \qquad (42a)$$

and

$$M_{3D} = -\,105 + 39.53\,E\,\epsilon_6 \qquad (42b)$$

All end moments may now be found in terms of ϵ_6 by proceeding backward and using the C-values and D-values in Eqs. 39 and 40; thus:

$$M_{2C} = -\,M_{3C} = -\,29.7 - 39.15\,E\,\epsilon_6 \qquad (43a)$$

and $M_{2B} + C_{2C}\,M_{2C} = D_{2C}$; or,

$$M_{2B} = -\,9.06 - 30.75\,E\,\epsilon_6 \qquad (43b)$$

At joint B, Eq. 31*a* should be applied to member 5, giving $\frac{C_{5B} + 2}{K_5} M_{5B} = \frac{1}{K_2} (2\,M_{2B} - M_{2C}) + 6\,E\,\delta_{5-2}$; or,

$$M_{5B} = +\,5.79 + 20.55\,E\,\epsilon_6 \qquad (44)$$

Continuing:

$$M_{1B} = -\,M_{2B} - M_{5B} = +\,3.27 + 10.20\,E\,\epsilon_6 \qquad (45a)$$

$M_{1A} + C_{1B}\,M_{1B} = D_{1B}$; or,

$$M_{1A} = +\,0.82 - 0.78\,E\,\epsilon_6 \qquad (45b)$$

and

$$M_{4A} = -\,M_{1A} = -\,0.82 + 0.78\,E\,\epsilon_6 \qquad (45c)$$

Step 4.—The transverse forces exerted on joints A and B by member 1 will be designated by F_{1A} and F_{1B}, respectively. Similar symbols will be selected for the transverse end forces of other members. The positive directions of these reactions will be such as to create a clockwise couple. Then:

$$\left.\begin{array}{l} F_{4E} = F_{4A} = -\,\dfrac{M_{4A}}{16}\,; \qquad F_{1A} = F_{1B} = -\,\dfrac{M_{1A} + M_{1B}}{10} \\ F_{5F} = F_{5B} = -\,\dfrac{M_{5B}}{16}\,; \qquad F_{2B} = F_{2C} = -\,\dfrac{M_{2B} + M_{2C}}{12.7} \\ \qquad\qquad F_{3C} = -\,\dfrac{M_{3C} + M_{3D} + 320}{16} \end{array}\right\} \qquad (46)$$

Load the conjugate frame with these forces, as shown in Fig. 13*b*, and equate to zero, the moment about point O of all forces on the free body above line X–X:

$$-\,10\,T_4 + 10\,F_{1A} - 9\,F_{4A} - 9\,F_{5B} + 6.35\,F_{2B} + 6.35\,F_{2C} - 9\,F_{3C} = 0 \qquad (47)$$

The force T_4 may be eliminated by the condition $\sum Y = 0$ at joint A, giving $T_4 = F_{1A}$. Use Eqs. 42 to 46, inclusive, and, in Eq. 47, replace all F-elements by their equivalents in terms of ϵ_6; the following equation is the result:

$$+ 318.67 + 224.51 \, E \, \epsilon_6 = 0 \quad \text{.................} (48)$$

or, $E \, \epsilon_6 = -1.41$.

The substitution of this value of ϵ_6 into the expressions of the several unknowns in terms of ϵ_6 will give all unknowns explicitly: For instance, $M_{3C} = + 29.7 + 39.15 \; (- 1.41) = - 26$; and $M_{3D} = - 105 + 39.53 \, (- 1.41) = - 161$.

All rotations and displacements of joints may now be found by using the end moments, the axial strains, and the unit deflections determined as a result of the preceding analysis. It is necessary, however, to use the actual K-values of the members; if member 4, for example, is a 10-in., 72-lb, H-section, its actual K is $\frac{420}{12^4 \times 16}$ ft^3, while its relative K was assumed as 1. Using feet and kips, the actual value of ϵ_6 (Eq. 48), is therefore: $\epsilon_6 = -\frac{1.41}{30{,}000 \times 12^2} \div \frac{420}{12^4 \times 16} = -\frac{1}{3{,}880}$.

If the vertical displacement of joint C is desired, Eq. 38*a* yields: δ_{3-9} or $\delta_3 = -\frac{2.22}{3{,}880}$. Then: $\Delta_3 = -\frac{2.22 \times 192}{3{,}880} = - 0.11$ in. (clockwise about joint D) or, $+ 0.11$ in. downward.

Conclusion

A complete analysis of the general two-dimensional framework is achieved through the consideration—and use as principal unknowns—of the system of the axial strains in the members and those in the imaginary diagonals which divide it into triangular elements. The paper brings into evidence the compatibility conditions and topological properties of the frame and establishes a correspondence between it and a conjugate frame which is statically determinate. These considerations lead to the determination of all unknowns.

The adaptability of the general theory to different types of problems, and its workability as a practical method of approach, are demonstrated by several illustrative applications.

Acknowledgment

This paper was prepared from a dissertation presented by the writer at Columbia University, New York, N. Y., in 1941, in partial fulfilment of the requirements for the degree of Doctor of Philosophy. The writer wishes to acknowledge his great indebtedness for valuable suggestions and guidance to Professors J. K. Finch and J. M. Garrelts, Members, Am. Soc. C. E., and R. D. Mindlin and M. G. Salvadori, Assoc. Members, Am. Soc. C. E., of the Faculty of the School of Engineering at Columbia University.

APPENDIX

Notation

The following symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering, and Testing Materials,[7] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. In the text, subscripts are used with these symbols, to relate them to the corresponding member or joint.

A = area; cross-sectional area of a member: A_e = the area of a quadrilateral element (Eq. 15); in Eq. 13, A, B, and C are used to designate the interior angles at joints A, B, and C of a triangle;

a = linear dimension in a triangle (see also b and h):

- a_1 = linear projection of the line AB (see Fig. 2) on the x-axis;
- $(a_1)'$ = linear projection of the line A'B' (see Fig. 2) on the x-axis;
- $a_{1,3}$, etc. (see Fig. 4) = linear projection of member 1 on member 3, etc.;

B = (see angles A, B, and C in Eq. 13);

b = linear dimension in a triangle (see also a and h):

- b_1 = linear projection of the line AB (Fig. 2) on the y-axis;
- $(b_1)'$ = linear projection of the line A'B' (Fig. 2) on the y-axis;

C = coefficient in the general linear relation between the end moments of a member; see Eqs. 26 (see also coefficient D); in Eq. 13, A, B, and C are used to designate the interior angles at joints A, B, and C of a triangle;

D = coefficient in the general linear relation between the end moments of a member; see Eqs. 26;

E = Young's modulus;

e = total axial deformation of a member, elongation being positive: In Eqs. 24, e_{xa} denotes the linear movement of support a in the x-direction and e_{ya} in the y-direction;

F = transverse end force in a member (also, where indicated, F = "function of"): F_{1A}, etc., in Eqs. 46, designate the transverse forces exerted by member 1 upon joint A, Fig. 13;

h = linear dimension of a triangle (see also a and b); in Eq. 9, $h_{1,2}$ is the perpendicular drawn on member 3 from the intersection of members 1 and 2;

I = moment of inertia;

K = stiffness ratio I/l;

l = length of a member from center to center of joints: l_1 is the length of member 1 = AB, Fig. 2; $(l_1)'$ is the corresponding length of the deformed member A'B', Fig. 2;

M = moment: M_{1A}, etc. = the moment exerted by member 1 upon joint A, the clockwise direction assumed positive; M_a = the moment at support a, Fig. 11;

[7] ASA—Z10a—1932.

m = moments similar to M when the end rotations and the lateral deflection of the member are zero;

n = a number:

$n_i = n_m + n_{mt} - 2\,n_p$;

n_m = total number of given members, excluding those connected to a support at each end;

n_p = total number of joints, excluding the supports;

n_r = total number of distinct regions in a frame, excluding those with all their vertices at supports;

n_t = total number of distinct triangles resulting from the division of the frame by imaginary diagonals, excluding those with all their vertices at supports;

n_{mt} = total number of imaginary diagonals required for the complete division of the frame into triangles, excluding those joining two supports;

n_σ = total number of linear movements of the supports in a frame;

T = axial force in a member, positive for tension:

T^0 = axial force found by assuming $X = 0$ and allowing the external loading to act (Fig. 8);

T^1 = axial force found by assuming $X = \dot{1}$ and all external loading removed (Fig. 8);

U = a substitution factor used to simplify Eq. 33;

X = reaction: X_a, Y_a, and M_a are the horizontal, vertical, and moment reactions at support a, Fig. 11;

x = coordinate distance parallel to the x-axis; x and y are the coordinates of a point on a deflected member; x_a, y_a are the coordinates of joint A;

Y = (see X);

y = (see x);

α = angles shown in Fig. 5;

Δ = relative lateral displacement at one end of a member with respect to the other, measured at right angles to the member, positive if either end moves clockwise about the other;

δ = unit deflection of a member, defined by $\delta = \frac{\Delta}{l}$; δ_t = unit deflection of an imaginary diagonal; $\delta_{p-q} = \delta_p - \delta_q$, the difference in unit deflection between two adjacent members p and q;

ϵ = axial strain of a given member defined by $\epsilon = \frac{e}{l}$; ϵ_t = axial strain in an imaginary diagonal; $\epsilon_{p-q} = \epsilon_p - \epsilon_q$, the difference in axial strain between two adjacent members p and q;

λ = the angle of a given line referred to the x-axis;

ω = angular rotation of a joint or support measured from its position before deformation or displacement, positive when clockwise;

$\sum_o$ = sign of summation around a closed region continuously in a clockwise direction, including all joints (or all members) on the boundary.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

ADVANCES IN SEWAGE TREATMENT AND PRESENT STATUS OF THE ART

SECOND PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SEWERAGE AND SEWAGE TREATMENT

The preceding report of the Committee on Sewerage and Sewage Treatment (1)[1] covered the period ending with the year 1941. With the onset of war in 1941 and the hurried preparations, the normal course of development in sewerage and sewage treatment was quickly interrupted. Civilian activities were greatly curtailed. Operation and maintenance became more difficult.

The number of sewage works in the United States increased from 5,580 in 1940 to 5,850 (2) in 1941. By the end of 1942 the total had risen to 6,199. In addition, during 1941–1942, more than 304 defense plant works were constructed and put in operation. By April, 1942 (3), many proposed projects for new sewerage works were shelved. However, urgently needed sanitation facilities were authorized in many areas where war activities had overwhelmed normal municipal functions.

The allocation to various types of treatment is shown in Table 1 (data from the 1940 census of the U. S. Public Health Service and the first supplement, 1941) for municipal and institutional sewage treatment works, exclusive of those at defense plants. A summary of the total number of municipal and institutional plants is given in Items 16, 17, and 18, Table 1.

Postwar Prospects

In view of all the delayed civilian work, the prospects appear excellent for the active construction of sewers, sewage treatment works, and garbage and waste disposal works in the postwar period. The *Engineering News-Record* estimates (4) this to amount to $217,842,000. Before the war, the reported volume of new sewerage construction reached its peak in 1939, with an annual

Note—Please forward all comments on this Report directly to Chairman Langdon Pearse, 910 South Michigan Ave., 7th Floor, Chicago, Ill. Progress Reports are published in *Proceedings* only.

[1] Numerals in parentheses, thus: (1), refer to corresponding references in the Bibliography (see Appendix).

total of $160,000,000, in contrast with a depression low of $22,000,000 in 1933. Further, considerable industrial waste treatment appears in prospect.

Sewage Research

The annual review of the literature on sewage and waste treatment and stream pollution by the Committee on Research of the Federation of Sewage Works Associations (5) continues to summarize the art each year from the research, laboratory, and operating standpoint. This Committee also listed 128 research projects (6) now under way in various localities throughout the United States. These are divided into four groups—sewage, industrial wastes, stream pollution, and methods. About one third the listings deal with industrial wastes. Thirty-one problems are further suggested for investigation.

TABLE 1.—Allocation of Various Types of Sewage Treatment Works

No.	Treatment	Municipal	Institutional
1	Imhoff tanks	2,337	238
	Separate Sludge Digestion:		
2	Total	1,380	102
3	Single-stage	1,287	99
4	Two-stage	93	3
5	Chemical precipitation	186	0
6	Guggenheim process	6	0
	Trickling Filters:		
7	Total	1,577	219
8	Conventional	1,358	205
9	High rate	219	14
	Activated Sludge:		
10	Total	328	35
11	Diffused air	136	2
12	Mechanical aeration	167	30
13	Not stated	25	3
14	Chlorination	1,055	163
15	Sludge gas in gas engines	91	0
	Totals:		
16	1940	5,060	568
17	1941	5,238	620
18	1942	6,199	

United States Public Health Service

The long-awaited report of the Ohio River Pollution Survey is completed and the factual data have been released in mimeographic form to a limited extent. The report to the U. S. Army Corps of Engineers, however, has not been released. This is being published as House Document No. 266, 78th Congress, First Session. The first supplement to the census of sewage works in the United States has also been issued. Research has continued on the biological and biochemical factors underlying the oxidation and assimilation of organic matters in liquids. The viability of certain ova and cysts in sewage and sludge has been studied. Special reports have been issued on waste resulting from munition manufacture.

Pollution Surveys

In the past two years a number of valuable pollution surveys have been completed or published. Some of the more prominent and voluminous are mentioned herein, because they indicate the foundation for a considerable amount of development and construction, mostly in postwar days.

Ohio River Pollution Survey

The final report (7) to the Ohio River Committee on the pollution of the Ohio River is one of the most elaborate and thorough surveys known. It is divided into three main volumes and six supplements. The main report covers not only the Ohio River, but the tributaries. In the supplements are discussed

the data on sources of pollution, organization and methods of laboratory studies, epidemiological and biological studies, acid mine drainage wastes, and industrial wastes. Supplement D on Industrial Wastes is an up-to-date treatise on various industries, and includes a new table on industrial equivalents.

Red River of the North

The report on the Red River of the North (8) shows that under an ice coverage the dissolved oxygen content is depleted to zero in from two to six weeks in the Red River. Due to natural pollution alone, the oxygen content of water in shallow lakes (less than 20 to 30 ft deep) may be depleted seriously, and even completely, during ice coverage (9). Large variations in stream loadings may occur because of sludge deposits, since the difference in stream loadings between two stations may be greater or less than the pollution loading between, depending on whether settling or scouring is taking place. Where neither occurs, the biochemical oxygen demand (B.O.D.) of the stream may be increased as the flow passes over sludge deposits.

Meramec River

The report (10) on the Meramec River Basin covers a watershed of about 3,980 sq miles in Missouri, extensively developed for recreation. The population is small (131,700 in 1940); hence the basin is relatively free of pollution. At no point was the dissolved oxygen content of the flow reduced to 50% saturation.

Androscoggin River

A report on the Androscoggin River (11) indicates that approximately 96% of the pollution reaching the main Androscoggin River is due to industrial wastes. Of the total pollution, 92% is from pulp and paper mills, and 71% is concentrated sulfite mill digester liquors which could be separated from other wastes. By proper utilization of the available oxygen resources of the river, generally objectionable conditions can be avoided.

Raritan River

Conditions in the Raritan River in New Jersey were studied by Rudolfs and Heukelekian (12) for three periods: The first (1927–1928), when practically no sewage or industrial waste treatment existed; the second (1937–1938), when practically all domestic sewage was treated; and the third (1940–1941), after some of the larger industries had also installed treatment facilities. The oxygen requirements at Bound Brook, N. J., doubled between the first and the third periods, producing a summer B.O.D. of about 12 ppm, with dissolved oxygen about 3 ppm or less. Sewage treatment improved the river somewhat from a public health standpoint, but further betterments are necessary.

Miscellaneous River Studies

Among the other stream pollution studies are those of the Tennessee River System (13) made by the Tennessee Valley Authority (TVA), and of the Roanoke, Va., Metropolitan District (14).

Odor Nuisance Studies in Lakes

An unusual study is the investigation (15) of the odor nuisance occurring in the Madison Lakes, particularly lakes Monona, Waubesa, and Kegonsa, from July, 1942, to July, 1943. The report shows the amounts of nutritive and pollutional material entering the lakes from various sources and indicates the inorganic nitrogen contributed from such sources and from deposits on the lake bottom is a critical element in the blooming of the lakes. A significant reduction in this nutritive material would reduce the frequency and density of lake blooms, but probably would not eliminate them or stop algae growths. The soluble phosphorus content of the water may also be a factor. The effluent of the Madison, Wis., sewage works contributes 79.6%. The nuisance is due to the excessive growth of algae and the odor created by it. It is significant, however, that prior to the construction of sanitary sewers in Madison, in 1885, the lakes bloomed. In 1882, a very vigorous blooming occurred, with scum sufficient to prevent boating near certain shores and creating an odor noticeable one or two blocks away (16). Subsequently, in 1919, Alvord (17) reported thereon, and in 1926 Domogalla (18) concluded the use of copper sulfate (30,000 lb per month in Lake Monona) was helpful, although algae were not entirely eliminated.

East Bay Cities Sewage Disposal Survey

On the Pacific Coast a detailed report was made (19) on the sewage disposal problems of the California cities of Alameda, Albany, Berkeley, Emeryville, Oakland, Piedmont, and Richmond. In San Francisco Bay the large areas of tidal flats near the sewer outlets of these municipalities present a different type of problem from that of Southern California with direct disposal into deep water of the Pacific Ocean.

Boston Harbor Pollution

On Massachusetts Bay, the pollution problems in Boston Harbor were studied in considerable detail in 1936 (20). This is supplemented by Weston (21) with data, particularly on the occurrence of grease.

Hampton Roads Sanitation Commission

The Hampton Roads Sanitation Commission was created by the Virginia Legislature in 1938, and includes the cities of Norfolk, Portsmouth, South Norfolk, Newport News, and Hampton and adjacent area. Under war conditions, the population has grown markedly. By 1955 a population of 451,000 is expected. A recent report (22) revises the general plan of sewage disposal, recommending five sewage treatment works, embodying settling and chlorination, separate digestion of sludge followed by vacuum filtration.

Miscellaneous Harbor Pollution

Among the miscellaneous harbor pollution studies are the reports (23) of the Interstate Sanitation Commission (Connecticut, New York, and New Jersey) and the report (24) on Everett Harbor, Wash., where 97% of the

pollution is due to waste sulfite liquor, which requires 395 tons of oxygen per day for stabilization. Concentration of waste in the water should not exceed 10 ppm of waste sulfite liquor solids.

Stream Pollution and Stream Standards

Stream pollution problems are discussed by Schroepfer (25) together with the various standards current in the United States, and, in particular, in relation to use, fish and aquatic life, agricultural use, and prevention of nuisance. This is supplemented by a Committee Report (26) classifying inland and shore waters in New England from the standpoint of highest use and locale, with the requirements for minimum quality of receiving waters according to use.

Ocean Outfalls—Pacific Coast

Since Warren and Rawn wrote "Disposal of Sewage into the Pacific Ocean" (27) and Rawn and Palmer indicated how to predetermine the extent of a sewage field in sea water (28), a number of sewage works have been undertaken in California, at San Diego and elsewhere. Most of the projects in Southern California differ from those on the Northern Atlantic seaboard by reason of having deep ocean disposal available, with moderate tides. Conditions around the Los Angeles outfall have grown steadily worse, with the rapid growth of tributary population (500,000 in 1920; 1,787,000 in 1940). As a result, a quarantine was established in 1943 on 10 miles of beach adjacent to the outfall in Santa Monica Bay at Hyperion. Gillespie (29) outlines the reasons which led the California State Board of Public Health to declare the condition a menace to health. In 1942 the sewage flow averaged 143 mgd, carrying about 160 tons of dry suspended solids into the Bay daily, including 8 to 10 tons of grease. The grease passes through the fine screens. It floats and is wind-blown. It clings to the bodies of bathers, some of whom take a "gasoline bath." Paratyphoid A and B were isolated within 200 or 300 ft of the outfall, in the surf, along the beach, and in the raw sewage. In a year some 20,000,000 people visited the 10-mile stretch, and on a busy day approximately 470,000 people. Nearly 40% are bathers.

As a limiting standard, 10 E. coli per cubic centimeter was used for assured safety for recreational use of surf water. An excess above the standard 20% of the time is allowed before deciding the portion of beach where the standard is not met.

Ocean Outfalls—Atlantic Coast

Since Weston wrote "Disposal of Sewage into the Atlantic Ocean" (30), progress has been made both in preliminary reports and construction. New York, N. Y., has built and placed in operation additional works, and is planning more. In the Norfolk and Hampton Roads (Va.) areas a number of smaller plants have been built.

Disposal of sewage by dispersion into coastal waters without preliminary treatment, other than coarse screening, was formerly the general practice in the New England area. In many cases, notably the Metropolitan Sewerage Districts of Boston, Mass., extensive surveys were made to select deep-water dispersal areas to obviate objectionable pollution of shore lines. Of recent

years there has been a definite trend toward supplementary treatment for the removal of sewage solids and grease.

From a deep-water outlet serving the South Essex Sewerage District, large quantities of grease balls, originating chiefly from tanneries in Peabody, Mass., were carried on to the beaches of Salem-Beverly Harbor and many miles beyond. As a result, grease separation tanks were installed on the outfall sewer, with simple, wide channels to recover grease by natural flotation and thus overcome the grease-ball problem in that area.

After periodical sanitary surveys of the waters and shores of Boston Harbor, sedimentation and chlorination are proposed to supplement the present method of disposal by dilution. Construction plans are under way for the South Metropolitan Sewerage District as a postwar project. Similar works will be provided for the North Metropolitan District and the Boston Main Drainage District.

The attitude toward disposal has changed in Massachusetts. In 1923, as a result of an investigation of the Merrimack River by the Massachusetts Department of Public Health, a trunk sewer some 36 miles long was proposed from Lowell, Mass., to an ocean outfall at the mouth of the Merrimack River. When recently reviewed, preliminary treatment by sedimentation was recommended for this project, largely to overcome objections of owners of shore properties in the vicinity of the proposed outfall.

Weston (21) indicates the need for grease removal in the partial treatment of sewage to improve the condition in Boston Harbor and its tidal estuaries, where some 244 mgd of sewage from a population of 1,840,000 enters through three different outlets. This sewage is of domestic origin, with a suspended-solids content during 1935–1936 of 0.13 to 0.28 lb per capita per 24 hr; 5-day B.O.D. of 0.13 to 0.28 lb per capita per 24 hr; ether-soluble matter of 0.04 to 0.09 lb per capita per 24 hr. Large sleek areas have formed. Grease balls have been observed 66 miles away. Proper settling and skimming apparatus will improve conditions and diminish the sleek areas. In experiments at Salem and Peabody, Mass., Weston found that treatment by 7-min settling, without air, produced practically the same results as with the use of aeration (0.18 cu ft per gal of sewage) for 7 min prior to 7-min sedimentation (see Table 2).

TABLE 2.—GREASE REMOVAL AT SALEM AND PEABODY, MASS.

No.	Operation	Type	Period (min)	Grease Removal: %	Grease Removal: Lb per million gal
1	Sedimentation	Fill and draw	7 to 10	57.9	400
	Aeration Plus Sedimentation:				
2	Aeration	Fill and draw	7	57.3	425
3	Sedimentation		7		
4	Sedimentation	Continuous	37	54	484

INTERSTATE COMPACTS

The first anti-stream pollution compacts to receive Congressional sanction were approved by the House of Representatives on July 1, 1941, after earlier

approval by the Senate. The first creates the "Ohio River Valley Water Sanitation District" and includes as signatory states Illinois, Indiana, Kentucky, New York, Ohio, Pennsylvania, Tennessee, and West Virginia. The second establishes the "Potomac Valley Conservancy District" and includes Maryland, West Virginia, Pennsylvania, Virginia, and the District of Columbia (31).

Interstate Sanitation Commission

The Interstate Sanitation Commission is a tri-state commission set up by compact by Connecticut, New Jersey, and New York to control pollution in their tidal waters. The compact was signed by New Jersey and New York in 1936. The Commission began to function in 1937. On September 17, 1941, Connecticut signed. The Commission has issued six annual reports (23), reviewing the conditions in the tidal waters and the betterments. The removal of suspended solids by sewage treatment plants increased from 39 tons per day in 1936 to 312 tons per day in 1941. The number of plants complying with the standards of purification set up by the compact have increased from 14% in 1937 to 63% (of 60 plants) in 1941. In 1942, out of 61 plants, 50% met the compact standards and 14 more almost qualified.

Effect of War Activity

Among the phenomena of war-time conditions is the change in the sewage of many communities, both from the standpoint of the content of B.O.D. and suspended solids and also from the standpoint of industrial wastes. This is reflected in large metropolitan areas. In The Sanitary District of Chicago, the human population is estimated to have decreased slightly since 1940, with a marked increase in the equivalent population (Table 3). In other localities

TABLE 3.—Equivalent Populations, The Sanitary District of Chicago

Year	Period reported	Population Equivalents		Human population
		Average	Maximum monthly	
1940	April through December	6,200,000	7,398,000	3,962,514[a]
1941	12 months	6,512,000	6,945,000	....
1942	12 months	6,693,500	7,755,700	....
1943	12 months	6,594,000	7,172,400	3,962,514[b]

[a] Census report. [b] Estimated; the population of Cook and Du Page counties in Illinois and Lake County in Indiana was estimated to have decreased 32,113 between April 1, 1940, and March, 1943 (according to the U. S. Census Bureau), from a total of 4,581,111 in 1940. Of this total, Cook County represents 4,063,342.

of smaller size, both the population and industrial wastes have doubled and trebled.

In the field of industrial wastes, the accelerated output of industry has increased materially the discharge of wastes to the stream. In many cases this has increased the B.O.D. load and also the amount of deleterious material. This is noted in the distilling industry in the Peoria, Ill., metropolitan area and the metallurgical wastes in Connecticut (32). The changes in industry have

frequently outstripped the ability to install remedial measures as in Michigan (33). In many cases temporary expedients have been provided. However, in many of the new plants constructed for war-material production, in critical situations, unusual attention has been paid to the elimination of industrial wastes such as the phenol and by-products wastes in an alloy steel plant, the neutralization of acid pickling liquors which might injure local sewers before reaching adequate dilution, and the treating of cyanide and chrome acid wastes from a large bomber plant (33).

In 1942, 48 treatment works were installed at the industrial plants in the United States, exclusive of those making munitions or connected with war operations (34).

Grease Recovery

Again war activities have stimulated attempts to recover grease, partly by drives to increase the collection from household sources and partly from industries or sewage treatment works (35). In World War I, grease was a problem in most army cantonments. In World War II—with the cooperation of the War Department, the Iowa Institute of Hydraulic Research, and various manufacturers—the problem was approached on the basis of adequate grease traps or interceptors installed at the source—that is, the sink. The efficiency of various devices or arrangement of baffling was first tested on a definite basis (35)(36).

Dawson states (36) that grease must be separated in a liquid state, by a reduction in velocity of the entering water by use of baffling and by preventing large scale turbulence. Well designed interceptors will maintain an average efficiency of 90% for a flow rate equal to 1 gal per min for each 1 to 1.5 gal of interceptor capacity. The grease retaining capacity should be equal, in pounds, to twice the flow rate in gallons per minute. Regular cleaning, at least once a week, is of prime importance.

From extended tests, specifications were prepared by the War and Navy departments for apparatus. For civilian use, ceramic traps are offered by various manufacturers.

The Chicago Section of the American Chemical Society appointed a committee which has been active in the preparation of recommendations for the War Production Board (WPB) on industrial fat recovery. The committee includes representatives from various large industries, as well as The Sanitary District of Chicago. The recovery of fats at the source has been vigorously stimulated, not only in the packing houses but in soybean extraction plants, vegetable oil refineries, soap making plants, and elsewhere.

Grease from Packing Houses

Mortenson (37) recommends that all fats reclaimable from packing-house wastes, be kept out of the sewers. In the wastes resulting from the slaughter operations and washing of raw product before processing, the fatty matter is non-emulsified and floats. This generally can be recovered with detention periods of 5 to 10 min and flow velocities of 3 to 6 ft per min. The settling solids must be promptly removed. Another type carries fatty matter originating in

the processing of by-products, cooking, rendering, etc. A large part of the grease is emulsified. The settling solids are relatively low in amount. A detention period of 1 to 2 hr, with flow velocities of 0.25 to 0.50 ft per min, is about the maximum which can be provided. Mortenson believes that the loss of recoverable fat can be kept to 0.1 lb per 1,000 lb of live weight for both slaughtering and processing, if adequate recovering basins are well operated. If the wastes are combined, the detention period for average flows is about 1 hr, with a velocity of 1 ft per min or less. (Those interested in the Mortenson paper (37) can secure reprints from the Industrial Fat Recovery Committee, WPB, 226 West Jackson Boulevard, Chicago, Ill.)

Grease from Sewage Works

In the treatment works of The Sanitary District of Chicago, two sources of material carrying grease were investigated. At the Racine Avenue Pumping Station (through which a large portion of the wastes of Packingtown pass) in 1942–1943 about 1,920 lb per day of skimmings were caught, yielding about 425 lb as grease in the laboratory. For the maximum month, this rate was about double. Such skimmings averaged about 40% water and 60% solids, of which 37% is ether-soluble. At the Southwest Works (which receives practically all the wastes of Packingtown), about 14,000 lb per day of scum accumulates with an ether-soluble content of about 50% of the wet weight. Of this amount, about 15% is unsaponifiable. The amount collected ranges from 5,000 to 20,000 lb per day. For these skimmings a refiner of grease pays 0.55¢ per lb, f.o.b. trucks at the works on a six months' contract. The same refiner pays 1.1¢ per lb for the skimmings at Racine Avenue on a two-year contract.

In New York City (38), the skimmings from four of its sewage treatment works have been sold at 0.8¢ per lb, f.o.b. the works, in containers furnished by the buyer. The estimated output is about 106,000 lb per month. About 75% or 80% of the dry weight of the scum is ether-soluble, of which 4.6% is unsaponifiable.

Analytical data on sewage scum in Chicago (Southwest Plant) and New York are as shown in Table 4(*a*). From preliminary data, the analysis of the ethyl-ether extract was as shown in Table 4(*b*).

Grease Recovery

The Des Moines, Iowa (39), sewage works renders its grease in an open tank heated by sewage gas burners. From 400 to 800 lb of brown grease are recovered daily and sold for 3¢ to 5¢ per lb. The plant serves a population of 120,000, with a flow of 12 mgd, and receives 1 mgd of sewage from five packing houses. At Denver, Colo., the skimmings yield 50% saponifiable fat, which contains 7½% glycerine (40). Owing to war conditions, the grease has increased from 1,000 to 2,500 lb per day. This is sold to a rendering plant (39) (41).

Gunson (40) states the grease from sewage scum may be an inferior grade because it contains a mixture of various animal fats, cooking oils, and soaps. However, after suitable treatment, it may serve as a source of glycerine and fat for low-grade soaps. Apparently not all refiners are equipped to handle such

scum. At Worcester, Mass., in 1942, 5,019 cu ft of grease were removed (42) from the settling compartments of the Imhoff tanks with a flow of 8,308 million gal. Attempts to use the grease failed, owing to the presence of mineral oil and fecal matter.

Grease in Sewage Treatment

Recently Fales and Greeley (43) discussed grease in sewage treatment, covering not only its characteristics, effects on sewage treatment plants, and

TABLE 4.—Grease from Sewage Works

Determination	New York, N. Y.	Chicago, Ill.
(a) Wet Scum		
Solids (percentage by weight)	31 to 59.4	....
Grease (percentage by weight)	68 to 81.6	55
(b) Ethyl-Ether Extract		
Ash (percentage by weight)	0.1	0.15
Melting point (degrees centigrade)	33 to 35	....
Glycerol, liberated chemically (percentage by weight)	4.0	3.0
Saponification value	192	170
Neutralization value	108	....
Unsaponifiable (percentage by weight)	4.6	13.0
Constants of the mixed fatty acids, prepared from paste:		
Iodine value (WIJ8)	54	61
Titre (degrees centigrade)	38	38
(c) All Other Matter (Percentage by Weight)		
Moisture	....	3.4
Impurities	....	0.15
Moisture, impurities, and unsaponifiable (MIU)	....	16.55

the tests for grease, but also the methods for removal and disposal. The Committee believes there is need for reform in writing about grease in the sewage art. Some authors distinguish between "oil" (that is, fuel and lubricating oil, gasoline, and kerosene) and "grease." Others use "fat" and "grease" as synonymous. Others assume that "ether-soluble" material is a satisfactory term, whereas in the laboratory various solvents (such as petroleum-ether, ethyl-ether, chloroform, isopropyl-ether, carbon tetrachloride, and benzine) are used, which may give very different results. The more common solvents are chloroform, petroleum-ether, and ethyl-ether. Of these, chloroform gives the highest results and petroleum-ether the lowest. Up to 1919, *Standard Methods* recommended ethyl-ether, but thereafter, petroleum-ether (44). Unfortunately, many, in quoting the determination of grease in sewage, have neglected to state the methods used.

Grease in Sewage

The grease content of various sewages reported (43)(45) in the last 25 years has varied chiefly between 25 ppm and 100 ppm, with a maximum of 500 ppm

in a sewage containing a large proportion of tannery wastes. On a per capita basis, the range is chiefly between 0.02 and 0.15 lb per day, with a maximum of 0.59 lb. The extent to which grease can be recovered from sewage varies over a wider range. In sludge digestion, grease is broken down. In the activated sludge process, it is oxidized. Thus both digested and activated sludge contain less grease than fresh sewage solids.

Because of the difficulty of determining the grease in sewage and the fact that practically all sewage works omit the determination from their routine, but little plant data are available. Gehm (46) offers a rapid method for determining grease in sewage and sludges by the use of mineral oil as an extractant. On the subject of grease in sewage is a general compilation by Mahlie (47); and special articles by Eliassen and Schulhoff (48); Gehm and Trubnick (49); Gehm (50)(51); and Heukelekian (52).

Waste Disposal Problems in Wartime

Mohlman (53) has reviewed the field of waste disposal, both at the sewage treatment works and in the industrial field, and finds that the industrial wastes have an equivalent population in the Ohio River watershed of 7,670,000; and in The Sanitary District of Chicago of 2,696,700. In the lower Illinois River during the summer of 1943, the increased discharge of distillery wastes in the Peoria-Pekin area (Ill.) upset the oxygen balance and practically wiped out the dissolved oxygen for 40 miles below Peoria. As the industrial tempo has risen, so have the wastes increased.

Further, Mohlman (54) has listed the war industries producing wastes under three general classes:

I.—Explosives and Munitions: Trinitrotoluol (TNT), trinitrophenol (picrates), smokeless powder, shell casings, and airplane motors and parts (aluminum and magnesium).
II.—New Industries: Synthetic rubber, alcohol, high-octane gasoline, food dehydration, and organic chemicals.
III.—Increased Production: Meat packing, and steel industry.

TNT Wastes

In the manufacture of TNT (54)(55), the waste waters are of two main types, one the acid or yellow water, and the other, alkaline or red water. These liquors are odorless and have no B.O.D., but both, and particularly the red waste, have intense colors which persist when greatly diluted. In many cases the color effect on the receiving waterway may be sufficiently controlled by decoloring with chlorine or by dilution, with storage of the wastes during periods of low stream flow. In extreme cases the only adequate answer may be evaporation of the wastes and incineration of the residue, which is very expensive.

The required dilutions were determined, which would not damage water supplies because of color and taste. Freedom from toxicity to fish and human beings and freedom from acidity occurred at a dilution of 1 to 10,000. If the water is not used for drinking purposes, a dilution of 1 to 3,000 is acceptable. The discovery that wastes from the manufacture of explosives such as TNT

were innocuous if sufficiently diluted has saved millions of dollars by obviating the widespread installation of expensive evaporators.

Other Munitions Manufacture

Smith and Walker have reported (55) the results of surveys made by the U. S. Public Health Service on wastes from the manufacture of TNT, smokeless powder, small arms ammunition, tetryl, and nitroglycerin.

Smokeless powder wastes have a large volume of liquid waste, are strongly acid, and high in sulfates and nitrate nitrogen. The wastes from various plants manufacturing small arms ammunition differ widely. On an average per 100,000 rounds there may be expected a flow of 36,000 gal containing about 100 lb of grease, 12 lb of copper, and 95 lb of volatile suspended solids, and a 5-day B.O.D. equivalent to 300 people. Tetryl wastes are acid and high in nitrates. With adequate dilution, no treatment should be required. The waste apparently is non-toxic. Nitroglycerine wastes show an intermittent acidity, but are low in organic matter and high in sulfates and soap hardness.

Distillery Wastes

Among the industries stimulated by war activities is the distillation of grain, in the endeavor to insure an adequate supply of ethyl alcohol, even though at a higher cost. The WPB 1943 quota for alcohol was 530,000,000 gal, of which 240,000,000 gal was to come from the distilled spirits industry, 225,000,000 gal from the industrial alcohol industry, and the remainder from the synthetic process using refinery by-products (53). This quota of 465,000,000 gal from fermentation processes necessarily requires the disposal of the still slop. According to the U. S. Public Health Service, 5,000 gal per day of 100-proof spirit (50% alcohol) produces still slop with a population equivalent of 60,000. The quota of 465,000,000 gal of 95% alcohol is equivalent to 2,400,000 gal per day of 100-proof spirit with a population equivalent of 28,800,000.

Among the available means of treatment are by-products recovery (including screens, concentration, evaporators, and dryers) (56) or anaerobic fermentation (57) with treatment of the digestor effluent on trickling filters. The feeding of slop to hogs or cattle has been largely discontinued. In some cases the suspended solids are removed by centrifuges prior to multiple effect evaporation followed by rotary driers. According to Black and Klassen (58), where grains are used a high-grade stock food can be recovered, amounting to 16 or 18 lb per bushel of grain ground.

Sulfite Liquor

Another war development is the production of alcohol from waste sulfite pulp liquor. The first successful plant in North America has been operated in Canada by the Ontario Paper Company, Ltd. (59), for more than a year. This handles more than 200,000 gal of waste sulfite liquor per day, and recovers about 18 gal of 190-proof alcohol per ton of pulp. With improved equipment a higher yield of 25 gal per ton of pulp is expected. A mill with a daily capacity of 100 tons of pulp is estimated to provide for an alcohol plant of economic size.

Operating costs (exclusive of overhead and amortization costs) show a range per gallon of 190-proof alcohol from 12.4¢ (for 200–250 tons of pulp daily) to 19.75¢ (for 60–80 tons of pulp daily). There are probably some 35 mills of more than 100 tons daily capacity in the United States, with a combined capacity of more than 6,700 tons of pulp per day. At an average of 18 gal of alcohol per ton of pulp, this is equivalent to about 40 million gal of 190-proof alcohol per year. With a few minor changes, Callahan estimates (59) that about 75% of the pollution problem can be eliminated from the sulfite pulp industry.

Sewage Disposal at Military Establishments

The problems of sewage treatment and disposal at the large army camps in general were met successfully. Treatment methods were chosen consistent with the needs and requirements of the locality, extending from plain sedimentation with chlorination to treatment by trickling filters (either low or high rate) and in some cases to activated sludge. High-rate trickling filters were employed in many of the large camps.

Since the original program of cantonment construction in 1940–1941, additional troop housing facilities and more extensive utilization of original facilities necessitated sewage treatment work extensions in a number of situations. Because sewage flows were reduced by water conservation measures below the original basis of design (occasionally to 40 gal per capita per day), treatment works extensions to care for increased troop populations have included only those treatment elements affected by the per capita loading of suspended solids and B.O.D., such as sludge digestion tanks and trickling filters. In one camp equipped with Imhoff tanks, separate sludge digestion capacity was added for secondary digestion and storage of partially digested Imhoff tank sludge. In another camp in the South having unheated digesters, the introduction of live steam has increased the digestion capacity. In some installations of low-rate trickling filters, facilities are provided for their conversion to high-rate filters.

In general, the design of military treatment works was influenced by shortages of critical materials. Plain concrete served for tanks where conventional design called for reinforced concrete. Substitutes were used even for items such as bronze-mounted valves and sluice gates. Occasionally wood was used for the construction of tanks. In some cases this practice was carried to extremes. At one camp, housing some 40,000 troops, the sewage treatment works consists of Imhoff tanks built with wood frame construction on a concrete foundation. Wood was used for the sludge draw-off pipes as well as the sluice gates and sludge draw-off gates.

Occasionally difficulty was experienced in the procurement of trained personnel and in the maintenance of adequate operating forces. At present, under the efficient organization of regional service commands, qualified army and civilian engineers are available, who are putting sewage treatment and disposal on a sound operating basis. Some of the most troublesome operating problems were found in the small plants which were designed, constructed and operated with scant technical knowledge of the problems of sewage disposal.

The popular misconception of the functions of the septic tank (which still persists) accounts for many of the difficulties experienced in such places.

In handling problems of sewage disposal, the Army has succeeded better than the Navy, on the whole. The Navy was slow to recognize the need for sanitary engineers, either for the design or operation of sewage treatment works. Recently, however, steps have been taken to add trained sanitary engineering personnel. In general, standards of sewage treatment are being improved at both Army and Navy establishments. Cooperation with state sanitary engineers is having a beneficial effect. The desire to excel may lead to over-refinement, with consequent waste of manpower and materials. If authorities fail to recognize the temporary nature of certain military establishments, improvements may be urged which hardly seem warranted during the present emergency.

Design of Army Sewerage and Sewage Disposal Works

In October, 1940, the War Department authorized its consulting engineers to prepare tentative bases of design for sewage treatment works for National Defense Projects (60)(61)(62). In the development, the characteristics of cantonment sewage flow were considered, as well as its strength and the probable temporary use of the works. The loadings were selected to meet climatic conditions. The general purpose (63) was to give security to public health and prevent serious nuisance. As the construction activities proceeded, the tentative standards were revised into more detailed form, for permanent use (64).

As a rule, the construction was of a fairly permanent character. However, in two cantonments (65), wooden tanks were used for clarifiers, built on concrete bottoms, with troughs and baffles of wood. Vitrified pipe was used where possible. The primary digester was built of reinforced concrete, the secondary of wood. The trickling filters lacked walls, the stone being placed on a grill of 2-in. by 4-in. redwood slats laid on a concrete bottom. This type of plant was inexpensive to enlarge and offered a relatively high salvage value. For the most temporary situations such demountable designs were urged (65).

The flexibility of two-stage high-rate biological filters is stressed by Eliassen (66), Porter (67), and others (68). Suitable emergency treatment is also recommended by Bachman (69).

Operation of Army Sewage Treatment Works

The release of data on the operation of Army sewage treatment works has been restricted. The outstanding articles by Kessler and Norgaard (70)(71) present many of the more important phases. The complete story will doubtless have to await the end of the war. Among the operating topics are: Grit, grease (6,000,000 lb per month), improperly placed rock in trickling filters, sludge digestion (handling of supernatant liquor by aeration or lime treatment before return), and contact aeration. Operating results show that in many projects the sewage flow was held to 70 gal per capita per day. However, Ellsworth found at Camp Edwards (Mass.) average flows of 80.7 and 90.5 gal per capita per day in the camp hospital and regimental area respectively (72). Analytical

data are given (71) on primary treatment, standard trickling filters, single and two-stage high-rate filters, activated sludge, and contact aerators.

Maintenance of Equipment

One feature of war conditions is the added emphasis on civilian maintenance of equipment and structures in sewage pumping stations and treatment works, such as the maintenance of electrical equipment (73), centrifugal pumps (74) (75), and a wide range of specific equipment (76). In Los Angeles County, to avoid the use of essential materials, manhole covers were made (77) of laminated wood strips. In other places pre-cast concrete, as well as terra cotta, frames and covers are also noted. Glass brick has been used at Chicago in replacing steel window frames and glazed sash.

Corrosion in Sewage Works

To reduce the corrosive action of hydrochloric acid on metal parts when cleaning filter cloths and vacuum filter parts at the Minneapolis-St. Paul Sewage Works (78), aniline oil is added as an inhibitor to the acid, in amount 2% by weight. With acid strengths up to 5% and with stronger acid, 3% or 4% of aniline oil, laboratory tests indicate that the corrosiveness of the acid is reduced 44%. In its vacuum filter installations, Minneapolis-St. Paul has utilized stainless steel alloy for valves and valve springs to resist corrosion.

For the protection of steel structures exposed to sewage and sludge, cathodic protection may prove helpful (79)(80). Corrosion troubles are listed by Reed (81) and by Schade (82) in a detailed committee report. Parkes (80) describes difficulties in a digester at Terminal Island (Los Angeles).

Industrial Sanitary Sewage Flows

For the sanitary flow per capita from industrial plants, some suggest 50 gal per capita per day. Smith and Walker (83) measured the flow over 24 hr from a war industry employing 6,000 persons, with the results shown in Table 5.

TABLE 5.—Sanitary Sewage Flows in War Industry

Shift	Midnight to 8:00 a.m.	8:00 a.m. to 4:00 p.m.	4:00 p.m. to midnight	24 hr
Flow (gal per capita)	31	20	19	22
B.O.D. (lb per capita)	0.061	0.040	0.040	0.044
Suspended solids (lb per capita)	0.086	0.063	0.053	0.064

These results are lower than those cited by the Army engineering manual (64), which allows 30 gal per capita with 0.10 lb B.O.D. and 0.13 lb suspended solids per capita for 8-hr shifts at plant, post, and storage projects.

From one year of operation of war plants, Klegerman (84) found:

Plant	Gal per capita per day
Plane assembly	23
Plane parts	13
Steel plate mill	20

Sharp peaks occurred at change of shift, reaching 300% of the average daily flow, for a short duration. At an airplane parts plant the per capita suspended solids were 0.017 lb and the B.O.D. was 0.035 lb.

Proportional Weirs

One of the earliest proportional weirs was installed by H. H. Sutro (85)(86) in a water treatment plant in 1908. This had an opening with one straight side and one curved. In 1914, Rettger (87) suggested a weir in which both sides were curved. With all such weirs the flow is intended to be directly proportional to the head. A Rettger weir was installed in 1915–1916 by the late John H. Gregory, M. Am. Soc. C. E. (88), in the grit chamber of the Albany, N. Y., sewage treatment works. Recently Smith (89) presented data on such weirs, which serve to maintain a constant velocity through tanks with large variations in flow.

Grit Chamber Design

Among recent designs for grit chambers, Camp (90) suggests the use of a standing wave flume to control the water level in a grit chamber of parabolic cross section and maintain a uniform velocity from 0.5 to 1.2 ft per sec with variable flow. The Sutro weir will control a rectangular grit chamber, but requires a head loss equal to full depth of the chamber. Camp proposes a chamber of approximately parabolic cross section followed by an adjustable control section, such as a Parshall flume, which provides a velocity control over a wide range of flows with a loss of head less than a Sutro weir. A design somewhat similar to that of Camp but with sloping side-walls was used in 1931 by Holmes (91) at Newark, N. Y. In 1932 at Mogden, England, Townend (92) placed a venturi flume at the outlet of each grit chamber, to control the water level and maintain a velocity of 1 ft per sec at all flows. The cross section of the grit chamber was a modified parabola.

An entirely different development was made by Fuller (93) at Olean, N. Y., following suggestions of Blunk and Edwards (94). The grit chamber itself is a hopper-bottom circular tank, with a central sump from which the grit is removed by a compressed air lift, with an upward velocity of about 10 ft per sec. Thus the grit is said to be removed and washed in one operation.

Sedimentation

The multiple-tray clarifier has again appeared (95) as a device for sedimentation. An installation (96) at Springfield, Mo., in 1940, consists of a series of trays, placed one above the other, each provided with a revolving radial arm that serves as a sludge scraper. Circular steel tanks were installed inside old rectangular tanks, with three trays in each tank. The original tanks removed 41.2% of the suspended solids at a flow of 3.5 mgd, a detention period of 2 hr, and an overflow rate of 700 gal per sq ft per 24 hr. The new tanks operate at nearly 10 mgd, with half the flow return sludge from intermediate and final clarifiers following trickling filters. The detention period is 33 min, the overflow rate 725 gal per sq ft per 24 hr. The removal of suspended solids is

reported to be 61%. The large increase in percentage removal may be due (97) to the flocculating effect of the returned sludge and the higher solid content in the influent.

The Gilchrist tray clarifier (98) was tested on mixed activated sludge liquor in 1933 at the North Side Works in Chicago. The apparatus was 10 ft in diameter, with 6 trays, having a total water depth of 12 ft. The results were not encouraging. A very low rate (286 gal per sq ft per tray per day) was required to approach the quality of the control effluent and actually around 150 gal per sq ft per day to equal. The control operated at rates from 1,255 to 1,818 gal per sq ft per day.

Activated Sludge

Activated sludge plants are still in favor where a high-grade effluent is required. At present there are at least 138 plants in the United States blowing air through diffusers. Among the more recent plants are those in New York City at Tallmans Island, Bowery Bay and the 26th Ward, as well as the projected Hunts Point plant. Thus the trend of the larger plants of New York City is toward activated sludge. Owing to the cessation of construction during the war, the West-Southwest Works (Chicago) are still incomplete. In the National Defense Projects, there are at least twenty-three activated sludge plants, but the type is not known. The number of plants in 1940, and those added in 1941 and 1942 are shown in Table 6, the schedule at the end of 1942

TABLE 6.—Activated Sludge Plants in the United States

At the end of:	Diffused Air			Mechanical Aeration			Type Not Stated		
	Municipal	Institutional	Military	Municipal	Institutional	Military	Municipal	Institutional	Military
1940[a]	130	2	....	150	25	....	16	2	....
1941[b]	6	....	....	17	5	....	3	....	....
1942[c]	....	....	....	....	....	....	6	1	23
Totals	136	2	0	167	30	0	25	3	23

[a] From U. S. Public Health Service. [b] From U. S. Public Health Service (first annual supplement). [c] From reference (34) in the Bibliography.

being indicated in the last line (99). The totals are as follows:

Municipal	328
Institutional	35
Military	23

Mechanical Aerators

For the smaller municipal and institutional plants, the mechanical aerators are in continued popularity. A considerable number were installed in war work. There are (99) more than 197 installations in the United States, of which 167 are municipal.

Sewage Aeration Tests

At the Wards Island Sewage Treatment Works, New York City (100), a pilot activated sludge plant has operated with a constant flow of 15 gal per min of primary settled sewage, since early in 1941, on three types of sewage aeration: (1) Sewage aeration without the return of activated sludge or liquor; (2) sewage aeration with the continuous return of fourth or last pass aeration tank liquor; (3) sewage aeration with the return of activated final settling tank sludge to maintain a mixed liquor suspended solids concentration between 150 and 500 ppm.

The results showed that for 1.5 hr of aeration a reduction of 30% to 40%, 60% to 65%, and 65% to 75% of the suspended solids and B.O.D. could be achieved, respectively, in operations 1, 2, and 3 previously stated. For 3 hr of aeration, suspended solids and B.O.D. reductions of 60% to 65%, 65% to 70%, and 75% to 85% were achieved, respectively, in operations 1, 2, and 3 previously stated. The returned sludge varied from 6% to 25% of the sewage flow.

Load Distribution in Activated Sludge Process

In 1937, Fair advanced the idea that incremental addition of sewage to returned activated sludge along the line of flow would be helpful. Later in 1941 McKee and Fair (101) indicated that load distribution of sewage into the aeration tanks in the activated sludge process would improve the operating efficiency by maintaining the oxygen demand at a uniform level. This would produce a sludge with good purifying capacity and settling properties, attain a decided saving in tank volume, and secure flexibility of operation to aid in the control of bulking and blanket rising and in the reduction of the possibility of shock by sudden discharges of toxic waste.

In practicing load distribution, the aeration units should provide pre-aeration of sewage and re-aeration of sludge prior to mixing; reduction of short circuiting and longitudinal mixing by suitable baffles; multiple-point introduction of sewage along the lines of flow. The sewage inlets should be equipped with flow controls.

At New York City, Gould independently developed along similar lines a so-called "step" aeration, which was first installed (102) at the Tallmans Island plant. With this arrangement he believes that a greater quantity of sewage can be treated with a shorter detention period and a lower air consumption than by the conventional method. The procedure is covered by U. S. Patent No. 2,337,384 (103).

Step Aeration

At Tallmans Island, New York City, two years of experience with step aeration has shown (104) that it is desirable to use both conventional and step aeration as conditions dictate.

Step aeration is used (the number of steps being dependent on the intensity of the conditions): (1) When there is persistent increase in the volatile content of the activated sludge—indicating the need for more oxidation of adsorbed organics; (2) when the sludge index (Donaldson) is decreasing steadily or dissolved oxygen is dropping steadily—which indicates the need for increased

sludge aeration period as the sludge is probably overloaded; (3) in case of high *Sphaerotilus* growths which are taken as an index of overloading or poor sludge condition.

Conventional aeration is used:

(1) When the primary effluent solids are low or the flow is low, thus requiring less solids in the aerator, but with the same end concentration, avoiding undernourished activated sludge.

(2) When there is over-aeration, as indicated by high dissolved oxygen and a pin-point floc in the final effluent which does not settle readily.

Since February, 1941, an experimental plant has operated at Wards Island with parallel comparison, using mixed liquor solids around 1,000 to 1,500 ppm. Preliminary results (October, 1941) indicate that, for aeration periods around 6 hours, the conventional is equal to or better than the step aeration. For sewage loadings to produce about 3-hr aeration, the conventional aeration reached a safe limit and was inferior to step aeration. The sewage load was increased in the step aeration process to produce a 1½-hr aeration period. After a breaking-in period, satisfactory effluents were obtained (with effluent B.O.D. seldom more than 20 ppm) during four weeks of operation.

In the Bowery Bay activated sludge plant (40-mgd design basis) in New York City (104), settled sewage is added in four separate increments to the aeration tanks while the return sludge from the final tanks is admitted at the head end of the first pass of each tank (two tanks, each with four passes). After the first few weeks of operation, the first pass was used for the re-aeration of the return sludge, and the settled sewage added at the end of the first, second, and third passes of each tank. Return sludge was held at about 40% of the sewage flow. The operating program was as shown in Table 7.

TABLE 7.—OPERATING PROGRAM FOR STEP AERATION (PPM)

Pass	Suspended solids	Dissolved oxygen
First	3,000 to 5,000	1
Final	1,000 to 1,800	3[a]

[a] End of pass.

However, the trial of stage addition of activated sludge at Two Rivers, Wis. (105), was inconclusive as to the value of such practice in improving the economy and capacity of activated sludge plants, owing to the inability to control the backflow of sludge in the aeration tanks and to allow for the addition of sludge in normal stages as originally planned. McKee and Fair (101) also noted the need of eliminating backflow conditions.

As step aeration is of considerable interest, the Committee hopes that enough data will soon be published of the various tests in New York City to show its merit.

SEDIMENTATION OF FINAL EFFLUENT FROM AERATION TANKS

The use of an effluent trough over the inlet to a final settling tank was first tried in the activated sludge tests at the Stockyards Testing Station (Chicago) in 1917. At Houston, Tex., in 1919, Fugate increased the rate of effective flow

through the final settling tanks of the activated sludge plant by installing outlets at the inlet end of the tanks. In both cases, no mechanisms were used in the tanks. At Houston the tanks had pyramidal hopper bottoms.

In 1931, at the Des Plaines River Works (Chicago), tests were made to determine the behavior of straight-line flights in a rectangular settling tank (about 68.6 ft long by 10.5 ft wide, 11 ft deep at the influent end and 10 ft deep at the effluent end) following the aeration tanks, as compared with the conventional hopper bottom tank without mechanism and a clarifier mechanism revolving in a square tank. The flight moved at about 1 ft per min toward the outlet end of the tank. In this study Zack noted eddy currents at the effluent end of the rectangular tank with single outlet weir. These currents carried sludge over the effluent weir. Various positions were tried for the effluent trough (double weirs) such as 13.33 ft back from the effluent end of the tank (with single and double troughs); troughs at 13.67, 34.44, 55.33 ft, respectively, from the effluent end; also running back both 40 ft and 50 ft from the effluent end.

In 1931–1932, Zack and Tolman made tests at the activated sludge plant in Springfield, Ill., in which two types of final settling tanks were installed—a square tank with a rotary mechanism (Dorr type), and a rectangular tank with a straight-line mechanism. The results indicated rather poor settling qualities. Moving the effluent weir back from the outlet end increased the removal of suspended solids in the Dorr tank, as did an H-weir placed in the rectangular tank.

At North Toronto, Ont., Canada, in 1934, the performance of the final tanks was greatly improved by locating two effluent troughs, respectively, about one quarter and three quarters of the distance from the inlet to the outlet ends of the tanks. The length of weir was increased per unit tank (65 ft square) from 65 ft to 240 ft.

In the activated sludge works at Chicago, the rotary clarifier mechanisms were installed in the final settling tanks—first at the North Side, in square tanks 77 ft square and 16 ft deep at the center, and later in circular units. In the square tanks the arms of the clarifier mechanism had movable extensions which swept the corners. There were three effluent troughs in the center of the tank. Mixed effluent was admitted from two opposite sides. This arrangement worked better than straight flow across the tank. Later the center-feed tank was adopted of circular form at the North Side extension and Southwest Works, and square at the Calumet Works.

TABLE 8.—SUSPENDED MATTER EFFLUENT (PPM)

Tank	Rate (Gal per Sq Ft per Day at Influent)			
	700	1,200	1,800	2,300
Round, new......	5.0	5.4	7.0	4.7
Square, original...	7.0	10.6	17.8	16[a]

[a] At the influent rate of 1,800 gal per sq ft per day.

At the North Side Works, the new tanks were 75 ft in diameter with peripheral overflow weirs. The tank depth was 14 ft 9 in. at the center. For a short period in July, 1937, comparative tests were made, with sludge having an index

between 74 and 83, which showed suspended matter in the effluent as indicated by Table 8.

At the Calumet Works the tanks are 91 ft square with peripheral overflow weirs. The tank depth is 15 ft 4½ in. at the center. At the Southwest Works the tanks are circular, 126 ft in diameter with peripheral weirs. The tank depth is 11 ft at the wall with a bottom sloping to the center with a pitch of 1 in. per ft. The settling qualities were much poorer than for the smaller units at the Calumet Works and North Side Works. After many tests (Table 9), an effluent trough was installed in one unit 14 ft in from the outer wall, which improved the settling qualities. Thus the Sanitary District has no rectangular final tanks equipped with straight-line flights.

TABLE 9.—FINAL SETTLING TANKS; SHORT TESTS UNDER SIMILAR OPERATING CONDITIONS, SOUTHWEST WORKS, THE SANITARY DISTRICT OF CHICAGO (Average of Five Tests)

Operating condition	Hr	Ft	Gal per Day per Sq Ft		Sludge index	Suspended Solids (ppm)			
	Duration each test	Depth to sludge blanket	Inflow	Over flow		Raw sewage	Mixed liquor (inflow)	Sludge return (underflow)	Effluent (overflow)
Standard tanks	5.6	11	1,120	860	52	214	3,290	14,600	18.2[a]
One additional effluent trough	5.6	11	1,120	860	52	214	3,290	15,900	13.8
Two additional effluent troughs	5.6	11	1,120	860	52	214	3,290	15,600	10.6

[a] Effluents of two tanks averaged for each test, except for one test, when only one was used.

The final settling tanks at the Bowery Bay Plant, New York City (106), are rectangular tanks 93 ft 5 in. long by 16 ft 10 in. wide, equipped with straight-line scraper mechanism with the bottom flights moving toward the sludge sump at the effluent end of the tank, where the sludge is withdrawn. The average tank depth is 12 ft 3 in. Gould states that the natural flow in that direction along the bottom (from 5 to 10 ft per min) aids in a much quicker removal of sludge. The original scraper was designed for a lineal velocity of 3 ft per min. This was later speeded up. A return flow occurs along the top of the tank toward the inlet end. The effluent weirs are placed along the sides of the tank for about 40% of the distances from the effluent end. In the last 8 months of 1942, the monthly average suspended solids in the effluent ranged from 7 to 18 ppm, averaging 12 ppm for the period. The final tank overflow varied from 774 to 960 gal per sq ft of tank surface per day. The sludge return ranged from 31% to 58% of the sewage flow.

Comparative operating data are shown (Table 10) for the final settling tanks at Chicago, Cleveland (107), and New York City. Yearly averages for three years are shown in Table 11 for the removal of suspended matter and settling rates at the North Side, Calumet, and Southwest activated sludge plants.

At Peoria, early in 1943, according to Longley, an effluent weir was installed near the inlet end of one of the final settling tanks. Comparative tests indicate

an average reduction in a 16-day test from a normal suspended matter content of 23 to 12 ppm in the effluent.

It is evident from the data available that activated sludge behaves differently from raw sewage solids in final settling basins. Eddy currents have a

TABLE 10.—COMPARATIVE TANK OPERATING DATA ON FINAL SETTLING TANKS IN ACTIVATED SLUDGE PLANTS IN 1940

No.	Item	The Sanitary District of Chicago			Cleveland Easterly Plant	New York City	
		Southwest Battery A	North Side	Calumet		Wards Island Plant	Bowery Bay[a]
1	Percentage volatile, raw sewage	70	67	63	68	81	....
	Suspended Solids (ppm):						
2	Raw sewage	203	144	121	236	220	197
3	Preliminary tank effluent	123	....	....	135	161	141
4	Sludge return	12,100	11,600	16,100	13,760	4,540	3,980
5	Mixed liquor	2,787	2,400	3,830	2,800	1,900	1,600
6	Final tank effluent	17	11	12	13	14	12
	5-Day B.O.D. (ppm):						
7	Raw sewage	156	129	107	130	207	145
8	Preliminary tank effluent	118	....	....	77	168	107
9	Final tank effluent	12	7	10	11	12	15
10	Sludge index	72	85	59	66	128	145
11	Settling rate, in gal per day per sq ft[b]	960	1,057	831	755	1,150	1,245
12	Solids to final tanks, in lb per sq ft per day	22.3	21	26.5	17.6	16	16.6
	Solids Removal (%):						
13	Mixed liquor	99.4	99.5	99.7	99.5	99.3	99.25
14	Raw sewage	91.6	92.4	90.1	94.5	94.0	93.9
15	Preliminary tank effluent	86.2	....	....	90.1	91.5	91.6
	5-Day B.O.D. Removal (%):						
16	Raw sewage	92.3	94.6	90.7	92.3	94.0	89.7
17	Preliminary tank effluent	89.8	....	....	86.0	93.0	86.0

[a] For period from May to December, 1942, inclusive (106). [b] Sewage flow plus return sludge.

TABLE 11.—REMOVAL OF SUSPENDED SOLIDS, ACTIVATED SLUDGE PLANTS OF THE SANITARY DISTRICT OF CHICAGO

Year	(a) Calumet			(b) North Side			(c) Southwest Sewage Treatment Works, Battery A			
	Suspended Solids (ppm)		Settling rate (gal per sq ft per day)	Suspended Solids (ppm)		Settling rate (gal per sq ft per day)	Suspended Solids (ppm)			Settling rate (gal per sq ft per day)
	Raw sewage	Final effluent		Raw sewage	Final effluent		Raw sewage	Preliminary effluent	Final effluent	
1940	121	12	831	144	11	1,057	203	123	17	960
1941	130	12	925	146	9	1,135	205	121	17	986
1942	129	12	1,013	139	10	1,070	205	127	14	813

considerable influence on its behavior. Only comparative tests using various arrangements will indicate the limits to which the modification of conventional designs can be carried. The arrangements may differ with the type of basin. Besides consideration of the area, depth, and shape of the tank, a thorough study of the other factors involved is needed, such as inlet velocities, velocity of approach to the outlet weirs, the length of weirs, and their location.

Maintenance of Diffuser Plates

At activated sludge plants using diffused air, the maintenance of diffusers is receiving increasing attention. Since this Committee commented (1) on this topic in its 1942 report, investigations have continued at Chicago, Cleveland, and Milwaukee. The Chicago studies have been on a broader scope to determine why the diffuser plates clog more rapidly at its Southwest and Calumet Works. The studies to date have verified the observations at Milwaukee on the increased resistance of diffuser plates when the atmosphere is very humid in summer. Of the methods tried for cleaning the plates, draining the tank and drying the plates is as effective as acid cleaning for surface conditions. However, in some three years of operation at the Southwest Works, clogging has developed in the plates from underneath, due to insufficient devices for cleaning the air. Grids of slotted pipe are being tried experimentally. To maintain the air supply temporarily, one row of diffuser plates with a porosity of 80 is to be installed. After the war new diffuser plates and more efficient devices for air cleaning will be provided. Undoubtedly the existing condition has been hastened by the high velocity of the air through the diffuser—that is, 3 to 4 ft per min, instead of the lower rates around 1 ft per min current in many plants. At Milwaukee an additional row of plates is being added, to make 3 rows along a channel 21.5 ft wide.

General methods on servicing porous air diffusers are described by Roe (107). In tests on cleaning diffuser plates at Chicago, the addition of chlorine to the air proved of no value. At Buffalo, N. Y., Symons reports (108) that the addition of chlorine to the air supply for 8 hours caused the surface growth to slough off. At Jackson, Mich., Jackson (109) states the use of chlorine increased the period of service between acid cleaning. Enslow (108) considers chlorine effective only on organic growths and of no value on plates clogged with inorganic deposit.

Reports from scattered plants vary (110). At Lima, Ohio, plates are cleaned once a week, and at Anderson, Ind., every 6 weeks using acid; at Muncie, Ind., from 2 to 12 months; whereas in Ohio at Mansfield and Columbus the first cleaning was given after 4 years of use. Denver reports 5 years and Richmond, Ind., 7 years of use, before cleaning. Diffuser tubes also require cleaning. At Gary, Ind., tubes were cleaned at intervals, the shortest interval being 1 month, the longest, 1 year. The tubes are cleaned once a year at Flint, Mich., and once every 18 months at Chatham, N. J.

A number of factors are concerned in the behavior of diffuser plates: (1) The porosity; (2) the rate of air flow per square foot per minute; (3) the amount of dirt in the atmosphere; (4) the efficiency of the air cleaning devices; and (5) the character of the sewage, particularly the presence of ferrous salts. From the standpoint of design, a question of economics is involved as to the probable life of plates of different porosities under various rates of flow of air per square foot on plate. There is also need of a simple, cheap container built for easy removal of the plates. The use of diffuser tubes does not as yet indicate a solution of the problem.

Actually little is known of the amount of dirt in the atmosphere at activated sludge plants. The only analyses known to the Committee are at the works of The Sanitary District of Chicago (see Table 12).

TABLE 12.—DUST IN THE AIR (MILLIGRAMS PER THOUSAND CUBIC FEET OF AIR) ACTIVATED SLUDGE WORKS OF THE SANITARY DISTRICT OF CHICAGO

Works	Period	Atmosphere	Filtered
Southwest	1941-1942	9.2	3.5
Calumet	May-October, 1936	5.3	2.89
North Side	January-April, 1934	2.98	1.67
North Side	May-June, 1934	2.58	1.15

At Cleveland (111) equipment is being perfected for the quick and efficient washing of plates after removal from the containers, and, if possible, adapted to wash plates in place. Considerable difficulty has been reported with the aluminum containers, apparently due to the corrosion of the tap bolts. A study (112) has also been made of the cost of diffuser plate cleaning as compared with compressed air cost.

TWO-STAGE AERATION

Two-stage aeration (113) has reappeared, in which the aeration tanks are in series or two stages, separated by an intermediate settling tank. This was first tried at Manchester, England (114), by Ardern in 1926. Extended tests (115) were made at the old Calumet Works (Chicago) between October, 1929, and October, 1931, on a flow of about 800,000 gal per day. The results indicated a satisfactory purification of a weak sewage with a relatively short aeration period (2.1 hours) or less than one half the period of a conventional single-stage spiral flow aeration tank. About 0.5 cu ft of air per gal was used, and about twice the settling tank capacity was required. The two-stage process was complicated to operate, very sensitive biologically, and required careful control. This confirmed the Manchester conclusions.

TRICKLING FILTERS

The conventional trickling filter is still in favor for sewage treatment of high grade. As of 1941, the U. S. Public Health Service census shows 1,487 municipal and 205 institutional installations, as compared with 1,388 municipal and 194 institutional installations in 1940. A few conventional filters were used in the army projects.

HIGH-RATE FILTERS

At the end of 1941, approximately 137 high-rate filter plants had been installed in the United States, of the Jenks and the Halvorson-Smith type. Since then the total has risen to 304 at the end of 1943.

Considerable data have been published on the design, construction, and operation of high-rate trickling filters of various types, licensed under patents issued to Halvorson-Smith (116) (known as aero-filters) or Jenks (117) (known as bio-filters) (117). Among the more complete statements are those of Fischer

(118) and of Montgomery (119)(120). There is also a third patent, issued to Ward (121). The Halvorson-Smith patent claims the uniform continuous distribution at a high rate. The Jenks patent claims the return of filter discharge to the raw sewage. The Ward patent claims the return of final effluent to the filter. The aero-filter practice does not recommend recirculation except to keep the minimum flow rate above 13 million gal per acre per day. The bio-filter practice recommends recirculation at all times, and according to conditions at times a combination of both filter effluent and final effluent or final sludge. In general the aero-filters are built of greater stone depth, from 5 to 8 ft, whereas the stone in the bio-filters is from 3 to 4 ft deep.

Recently under the auspices of the Board of State Health Commissioners of the Upper Mississippi River Basin Sanitation agreement tests were made at nine plants in Iowa, Illinois, Minnesota, and Wisconsin (122). The loadings are summarized in Table 13. The conclusions were that single-stage plants may reduce the B.O.D. between 75% and 85%. Recirculation increases the degree of treatment principally as a multiple-stage filtration. The average for plants tested was slightly under 80%. Two-stage plants may reduce the B.O.D. from 84% to 95%. The average for plants tested was 90.2%. However, the B.O.D. of the final effluents from high-rate filters is rarely below 25 or 30 ppm.

The factors of primary importance affecting the B.O.D. reduction of sewage by high-rate filters and clarifiers are: (1) Preliminary treatment of sewage; (2) biological condition of filter; (3) filter loading expressed in pounds of B.O.D. per unit of filter surface per day; and (4) recirculation. Other variables such as the temperature of filter influent, the forced or natural draft circulation of air, and the dosing rate were considered relatively unimportant in these tests.

Filters should be designed on the basis of actual load applied. When recirculation is applied, the total flow and its strength should be used for computation. With 5-day B.O.D. loadings from 0.2 to 1.0 lb per sq ft of filter surface per day, approximately 63% reduction was obtained by filtration and clarification with filters 6 to 8 ft deep. A maximum removal of 0.7 lb per sq ft per day is indicated for all loading in excess of 1.2 lb per sq ft per day.

The reduction of suspended solids was almost 80% for single-stage and 90% for two-stage filtration.

High-rate filtration results in the reduction of organic nitrogen compounds but shows little oxidation of the ammonia. Settled effluents show little, if any, nitrate and nitrate nitrogen.

Uniform instantaneous dosing appears desirable. But no correlation was found between the distribution of sewage over the filter surface and the efficiency of treatment.

High-Rate Biological Sewage Treamtent

Greeley (123) presents a comparison of high-rate biological treatment processes, such as trickling filters, activated sludge and contact aerators (Hays process). He states that standard trickling filters are often operated at an applied loading of 300 to 400 lb of B.O.D. per acre-ft per 24 hr, and occasionally as high as 800 lb (Decatur, Ill., and Fort Worth, Tex.) or more. High-rate filters operate with loads ranging from 5,000 to 15,000 or 20,000 lb B.O.D. per

TABLE 13.—Summary of Loadings on High-Rate Filters
(Averages of 24-Hr Runs)

Plant	Number of runs	Recirculation ratio	Dosing rate (million gal per acre per day)	Biochemical Oxygen Demand: Lb per cu yd per day: Loading	Lb per cu yd per day: Reduction	Lb per acre-ft per day: Loading	Lb per acre-ft per day: Reduction	Depth of stone (ft)
(a) First-Stage Filter and Clarifier								
Glenwood City, Wis.	3	1.95	14.3	1.78	0.93	2,871	1,500	7
	3	2.91	15.9	1.35	0.89	2,178	1,436	
	3	2.08	13.0	1.27	0.80	2,049	1,290	
	3	1.63	10.6	1.06	0.54	1,710	871	
	3	0.00	4.1	0.77	0.42	1,242	677	
River Falls, Wis.	3	0.00	17.3	2.92	2.14	4,710	3,452	8
Owatonna, Minn.	3	0.84	16.8	1.37	1.05	2,210	1,694	7
	1	0.66	21.0	16.2	3.29	26,131	5,307	
	2	0.74	18.3	4.49	2.55	7,242	4,113	
	3	0.59	19.2	2.06	1.22	3,323	1,968	
Owatonna, Unit 1	3	0.00	6.7	0.90	0.70	1,452	1,129	7
	3	0.00	18.5	2.78	1.60	4,484	2,581	
Owatonna, Unit 2	3	0.00	7.1	1.12	0.65	1,807	1,048	6
	3	0.00	15.4	2.70	1.28	4,355	2,065	
Lakefield, Minn.	3	0.26	7.1	1.46	1.12	2,355	1,807	7.5
Austin, Minn.	3	0.00	21.4	7.38	3.22	11,904	5,194	6
Lake Mills, Iowa	3	0.17	12.7	0.90	0.58	1,452	936	7
Webster City, Iowa	2	0.44	18.8	1.91	1.04	3,081	1,678	7.5
Webster City, Unit 1	3	1.33	16.3	1.13	0.71	1,823	1,145	7.5
	3	0.83	15.1	0.51	0.25	823	403	
Webster City, Unit 2	3	1.33	16.3	1.13	0.64	1,823	1,032	7.5
	3	0.83	15.1	0.51	0.26	823	419	
Carlyle, Ill.	3	0.27	27.6	3.86	2.32	6,226	3,742	8
Paris, Ill.	3	0.27	15.1	0.58	0.32	936	484	6.25
(b) Second-Stage Filter and Clarifier								
Owatonna, Minn.	3	0.75	17.7	0.404	0.053	649	85	6
	1	0.58	21.5	16.28	1.296	26,281	2,091	
	2	0.58	18.5	2.312	1.332	3,739	2,149	
	2	0.64	20.5	0.899	0.440	1,452	709	
Lakefield, Minn.	3	0.00	5.6	0.279	0.172	450	277	7.15
Webster City, Iowa	2	0.44	18.8	0.865	0.382	1,397	617	7.5
Austin, Minn. (low-rate filter)	3	0.00	4.0	0.560	0.463	904	747	8.5

TABLE 14.—Summary of Operating Data for High-Rate Aeration Plants, High-Rate Biological Sewage Treatment

No.	Item	Chicago, North Side	New York, Bowery Bay	Hays process
1	Displacement period in aeration tanks (hours)	3.0	2.6	1.6
2	Return sludge (percentage of sewage flow)	21.7	46.0	0.0
	5-Day B.O.D. (ppm):			
3	Raw sewage	109	156	138
4	Preliminary sedimentation tank effluent	....	110	72
5	Final effluent	10	16	13
6	Percentage removal of B.O.D. by aeration and final sedimentation tanks	91	85	82
7	B.O.D. load applied to aeration tanks (pounds per day per thousand cubic feet of aeration tanks)	50.0	47.5	66.4
8	Air (cubic feet per gallon of sewage)	0.37	0.59	1.29
9	B.O.D. in raw sewage (pounds per capita per day)	0.19	0.16	0.17
10	Period covered	12 months (1936)	May, 1942, and 1943	March to August, 1943

acre-ft per 24 hr, but with low removals. With loads around 3,000 lb of B.O.D. per acre foot per 24 hr removals of 75% are attained. The activated sludge process may also be operated with a high rate—that is, a short aeration period—as shown at Chicago and Bowery Bay in New York City.

Comparison of the Chicago and New York plants with the Hays process in Table 14 shows a high use of air in the latter. Many of the Hays plants cited by Kessler and Norgaard (71) used air amounting to 1.25 to 4.0 cu ft per gal of sewage. The relation of air use to the pounds of applied B.O.D. is as follows (123):

Plant	Lb of B.O.D. applied per 1,000 cu ft of air
North Side	2.68
Bowery Bay	1.61
Hays process	0.47

Treatment of Sewage Diluted with Trickling Filter Effluent

At Rotherham, England (124), beginning in 1935, the effluent of trickling filters (high in nitrates) was fed into the aeration tanks along with the effluent of the primary sedimentation tanks, thereby increasing the capacity of the bio-aeration plant. Half the primary tank effluent goes to the trickling filter, and is used to dilute the other half.

At Sheffield, England, Edmondson and Goodrich (125) have been operating a high-rate trickling filter (so-called cyclo-nitrifying filter) in circuit with an activated sludge plant (Haworth type), through which a constant volume of plant effluent is continuously circulated. Laboratory tests led to a demonstration for 20 months with an actual filter 30 ft in diameter and 7½ ft deep, containing 196 cu yd of graded material ranging from ½ to ¾ in. in the top 18 in. down to 3 to 6 in. in the bottom 18 in. This was dosed at the rate around 900 imperial gal (1,125 U. S. gal) per cu yd furnishing 180,000 imperial gal (225,000 U. S. gal) per day of nitrified effluent, containing at first around 10 ppm of nitrate. The nitrate was increased to 20 ppm by improved distribution in the summer. Mixed volume for volume, an improved effluent resulted, lower in B.O.D. and containing more nitrate. The activated sludge was free from any bulking tendency.

Magnetite Effluent Filters

Effluent filters of crushed magnetite, arranged for cleaning by means of solenoids, have been abandoned at Denver, after 6 years of effort to remodel the apparatus. Because of mechanical difficulties uninterrupted operation could not be secured for even 30 days. The principal troubles were: Difficulty of insulating the solenoid windings so that the underwater service could be maintained, difficulty in maintaining a filter bed of uniform thickness, corrosion of the filter screen and its supports, and loss of filter material. During the 6 years the actual efficiency of the filters could not be determined because of the short periods of operation. The power usage was excessive. The removal of solids was relatively expensive.

At Cleveland in the Southerly plant, Ellms also reports a high cost of operation and upkeep, with difficulties similar to those at Denver, except for the

insulation of the solenoid windings. Little benefit was indicated in filtering the trickling filter effluent.

At Minneapolis-St. Paul (126), the effluent filters were in service for only a few days each year from July 6, 1938 (for example, 39 days in 1942). The erosion and displacement of the sand on the beds proved a major difficulty. The steel grating supporting the sand corroded as at Denver.

The removals obtained at Minneapolis-St. Paul on settled sewage were as shown in Table 15.

TABLE 15.—OBSERVATIONS ON MAGNETITE EFFLUENT FILTERS AT MINNEAPOLIS-ST. PAUL

(Parts per Million)

Season year	No. of days	B.O.D.		Suspended Solids	
		Influent	Effluent	Influent	Effluent
1940	105[a]	122	105	76	54
1941	178[b]	115	100	81	58
1942	39	150	140	99	80

[a] In 1940 filter operated 105, 24-hr days and 73 fractional days. [b] In 1941 filter operated 175, 24-hr days and 46 fractional days.

During 1937–1939 tests were made on magnetite filters at the West Side Works in Chicago, with flows from 1.40 to 2.27 mgd, and rates from 2.10 to 4.83 gal per sq ft per min on settled sewage, and with flows from 1.40 to 2.59 mgd and rates from 2.10 to 9.03 gal per sq ft per min on chemical precipitation effluent. Difficulty was experienced with the movement of the sand and its loss through the supporting screen. To replace the sand yearly would require about 60% of the original amount.

For filter rates between 2.4 and 2.75 gal per sq ft per min (at which most of the tests were run) the results were as given in Table 16.

TABLE 16.—REMOVALS OF TOTAL SUSPENDED SOLIDS AND 5-DAY B.O.D. BY MAGNETITE FILTERS

Description	Parts per Million		Percentage Removal[a]	
	Suspended solids	5-day B.O.D.	Suspended solids	5-day B.O.D.
Plain Settled Sewage:				
Filter influent	68	59	....	....
Filter effluent	47	45	....	....
Removed by Filtration:				
Minimum	11	5	8.7	5.4
Maximum	34	24	26.8	29.0
Average	21	14	17.0	15.3
Chemically Precipitated Sewage:				
Filter influent	48	39	....	....
Filter effluent	30	30	....	....
Removed by Filtration:				
Minimum	7	4	5.9	4.7
Maximum	32	19	25.9	19.8
Average	18	9	13.5	9.9

[a] Based on raw sewage.

Earlier tests at the North Side Works in Chicago, on the activated sludge final effluent indicated so little improvement that additional final settling tanks were decided to be more effective and economical for that plant.

Sludge Digestion

Rudolfs and Logan (127) report that digested sludge concentrates best at a temperature of 55° C. The rate of compacting in a given time is highest with the lowest initial concentrations. This rate gradually decreased as the initial solids concentration increased. The compacting of digested sludge was affected by time, the initial concentration, and temperature.

In coagulating supernatant sludge liquor Rudolfs and Faulk (128) obtained the best results by using lime in solution and ferric chloride. When 4.95 lb of lime as CaO in solution and 4.15 lb of ferric chloride were used per 1,000 gal of sludge, a 63.0% reduction in oxygen consumed resulted. Lime in solution was as effective as ferric chloride and more effective than lime slurry, in clarification and in the removal of oxygen-consuming substances. Calcium hydrate was better than dolomitic or high magnesium hydrates. For equal clarification and removal of oxygen-consuming materials, lime in solution was cheapest. At Princeton, N. J., supernatant sludge liquor is pumped (129) into a conditioning tank for treatment. If the solid content of the sludge liquor is high, alum is added and the clear effluent is discharged into the raw sewage or into the trickling filter influent. Iron salts, when used to reduce odors, are believed to aid sludge digestion.

At the Joint Disposal Plant of the Los Angeles County Sanitation Districts (130), steam is added to the raw sludge as it is pumped into the digester. Applied as steam, 1 gal of water will raise the temperature of more than 100 gal of wet sludge 10° F. Although sludge has been heated with steam for a number of years at Birmingham, England, this procedure is an innovation in the United States. Wittwer (131) discusses the general theory and practice of sewage sludge digestion tank heating.

TABLE 17.—Sludge Digestion; Westerly Sewage Works, Cleveland, Ohio

Type of material	Solids (%)	Volatile solids dry basis (%)	pH	Alkalinity as $CaCO_3$ (ppm)
Raw sludge	6.83	65.6	6.5	940
Digested sludge	7.48	48.9	7.3	3,380
Supernatant liquor	0.48	49.3	7.3	3,160

At the Westerly Sewage Works in Cleveland the primary sludge from Imhoff tanks is digested in the six 50-ft-diameter tanks (132)(133). The average analysis of the raw and the digested sludge for the 5-yr period from 1937 to 1941, inclusive (132), is as given in Table 17.

The average daily operating load amounted to 0.097 lb of dry solids per cubic foot of tank capacity and 0.064 lb of volatile solids. The reduction in weight of total solids was 33.75% and of volatile solids 50.62%. Gas production amounted to 8.92 cu ft per lb of volatile solids added and 17.63 cu ft per lb of volatile solids destroyed.

Sludge Digestion Tanks

In the field of sludge digestion, floating covers are still popular, some 73 plants having been equipped therewith on war projects. Up to the end of 1943 a total of 206 plants had been so equipped in the United States.

At Ludlow, Mass., Taylor (134) used a novel design of sludge digestion tanks. Above each primary and secondary digester there is a covered compartment 3 ft deep for the storage of excess supernatant liquor. Sludge flows from the primary into the secondary digester. Sludge liquor from the secondary digester is displaced into the upper compartments of each tank. Under normal operating conditions a negative pressure cannot develop in the tanks. The upper compartment also helps to insulate the sludge in the lower compartment.

Where small plants have fixed concrete covers on the digesters with the overflow designed to maintain a hydrostatic head under the slab of 15 to 18 in. for scum submergence, the cracking or lifting of the cover sometimes occurs by uplift. Modification of the overflow outlet is helpful for deep extension into the digester with an overflow elevation higher than the normal outlet, which affords a relief.

Vacuum Filtration of Sludges

In a treatise on the vacuum filtration of sludge Genter (135) indicates the fundamental factors affecting filtration, including the specific gravity of the sludge, filtration pressure, nature of sludge solids, water content, alkalinity, volatile content and temperature of the sludge, the kind and amount of conditioning agents, the method of mixing the sludge and the coagulant, the filter submergence, and the filter drum speed. Empirical equations have been suggested for the addition of coagulates to sludge to be dewatered by vacuum filtration. However, the best operating control is by laboratory checks with Buchner funnels and by careful observations during actual operation.

From the past 2 years available data on the performance of vacuum filters in sludge dewatering are summarized for raw primary sludge (Table 18(*a*)), digested sludge (Table 18(*b*)), and raw activated sludge (Table 18(*c*)).

The principal effort of operators during the war period has been to keep the filters in operation. Because of the difficulty of obtaining corrosion-resisting metals, the drainage piping of vacuum filters at Chicago has been replaced by plastics. A wooden grid has been substituted for the standard copper drainage screen under the filter cloth and the spiral winding wire is being discarded. The modifications in vacuum filter construction under test at Chicago were stimulated by the necessity of finding substitutes for essential material. Some of these modifications may become standard practice after the war. Wooden drainage panels are built up for the backing. This is divided into 24 panels in the large 11-ft 6-in. by 16-ft filters. A groove is left between each panel into which the cloth is crimped by a wooden spine nailed down. This permits the use of narrower cloths (instead of a blanket all in one piece) which can be purchased at a lower price. Both the spiral winding wire and the drainage mesh can be omitted. A floating scraper is being tried instead of a fixed rigid one. In the internal drainage system the original copper pipes with bronze fittings have corroded, and have been replaced by copper and brass; synthane, Saran; or synthane piping with glass elbows. Corrosion is greatest in the fittings, particularly the elbows. Three lengths of 4-in. rubber-lined vacuum piping were replaced due to collapse of the rubber lining.

Table 18.—Performance of Vacuum Filter Installations

Location	Year	Solids in wet sludge (%)	Conditioner Used		Cake		Bibliography reference
			Kind	Lb per 100 lb of dry solids	Lb of dry solids per hr per ft of filter area	Moisture (%)	
(a) Raw Primary Sludge							
Auburn, N. Y.	....	6.13	$FeCl_3$ CaO	4.35 13.46	2.81	66.6	136
Minneapolis-St. Paul, Minn.	1942	9.63	$FeCl_3$ CaO	1.20 3.44	3.40	66.3	137
Neenah-Menasha, Wis.	1940	10±	CaO	3.70	4.11	68.0	138
Detroit, Mich.	1941	....	$FeCl_3$ CaO	2.6 7.7	7.0	....	....
(b) Digested Primary Sludge							
Annapolis, Md.	1941	9.0	$Fe_2(SO_4)_3$	2.86	7.5	....	139
Baltimore, Md.	1942	4.9	Chl.$FeSO_4$	5.98	4.49	76.4	....
Birmingham, Mich.	....	....	$FeCl_3$ CaO	2.2 9.8	10.0	75.0	140
Buffalo, N. Y.	1940 1941	9.11	$FeCl_3$ CaO	2.93 11.21	6.77	63.1	141
Cleveland, Ohio:							
Southerly[a]	1940	6.0	$FeCl_3$ CaO	3.90 13.6	4.3	74.1	142
Westerly	1940	7.48	$FeCl_3$ CaO	3.01 12.60	3.74	66.3	143
Cortland, N. Y.	....	10.1	$FeCl_3$ CaO	1.68 8.98	9.51	57.7	136
Detroit, Mich.	1941	....	$FeCl_3$ CaO	2.6 9.6	6.0	....	....
	1942	....	$FeCl_3$ CaO	2.7 8.3	6.8	68.8	....
Hartford, Conn.	1942	8.6	$FeCl_3$	2.41	5.61	59.8	144
Lansing, Mich.[b]	1938 1941	9.1	$FeCl_3$ CaO	4.4 11.1	5.58	72.0	145
Marion, Ind.	1940	4.97	$FeCl_3$ CaO	2.95 14.83[c]	7.3	74.2	146
Muskegon, Mich.	1940	6.86	$FeCl_3$ CaO	2.9 7.9	6.2	72.0	147
San Francisco, Calif.	1940	3.5	$FeCl_3$	4.55	5.55	75.7	148
Springfield, Mass.	1941	11.4	$FeCl_3$	1.63	5.0	63.5	149
Washington, D. C.	1940 1941	6.5	$FeCl_3$	3.18	5.9	71.1	139
Winnipeg, Canada	1941	10.0	$FeCl_3$	2.6	4.2	68.8	139
(c) Raw Activated Sludge							
Chicago, Ill.:							
Calumet	1940	2.38	$FeCl_3$	7.6	1.97	81.8	150
Southwest	1942	2.02	$FeCl_3$	10.4	1.32	83.1	....
Houston, Tex.	1941 1942	1.26	Ferric sulfate	17.4	0.67	81.7	....
Milwaukee, Wis.	1941 1942	2.33	$FeCl_3$	5.58	1.84	83.12	....

[a] Sludge is a mixture containing activated sludge from the Easterly Works, as well as digested primary solids. [b] Sludge contains garbage. [c] 72.0% CaO.

Sludge Dewatering on Drying Beds

As of 1941 (99) the total number of municipal sludge drying beds in the United States was 3,570, of which 313 were covered. There was also a total of 339 institutional beds of which 35 were covered.

Sludge Disposal

During 1942–1943 the developments in sludge disposal were limited by the cessation of municipal construction as a wartime measure. The municipal mechanical dewatering installations increased from 61 in 1939 to 91 in 1941. The number of incinerator installations increased from 37 in 1941 to 42 in 1943. In 1941 there were 10 installations using the flash-drying system and 27 installations using the multiple-hearth system. At the end of 1943, there were 13 using the flash-drying systems and 28 using the multiple-hearth system. One of each type is not yet in operation. Where a heat-dried sludge is desired for fertilizer, the flash-drying system apparently still leads the field. Of the multiple-hearth type, 3 plants are handling mixed garbage and sewage sludge and one plant (Piqua, Ohio) is burning unfiltered liquid sludge. Experience indicates problems of maintenance with each type of apparatus. Doubtless by the end of the war various improvements in each type will be available. The problems and practice in dewatering and incineration of sludge are covered in extensive detail by a Symposium presented before the Sanitary Engineering Division of the Society (151).

Treatment of Excess Activated Sludge

In 1941 among the 357 activated sludge plants in the United States, 318 were equipped with separator sludge-digestion tanks. In 1943, 4 plants are dewatering and heat-drying sludge for sale as fertilizer—Chicago (2 plants), Houston, and Milwaukee. The methods of sludge handling as of the end of 1941 are as follows (U. S. Public Health Service Census):

Methods of sludge handling	Municipal	Institutional
Separate digestion	286	32
Digested in Imhoff tanks	6	1
Lagooning	6	..
Mechanical sludge dewatering	4	..
Incineration	2	..
Dewatering-drying	5	..
Sludge pumped to another plant	3	..
Barged to sea	1	..
Unknown	9	2
Total	322	35
Two-stage digestion units	22	0

The 322 municipal plants include three of the Guggenheim type—New Britain, Conn., Anderson, Ind., and Elwood, Ind.

Sludge Lagoons

For many years sludge lagoons were regarded as a temporary measure and a possible source of odors. Apparently the earlier difficulties have been surmounted by better operating control, for in 1941 the U. S. Public Health Service Census shows 73 municipal plants (distributed among 23 states) utilizing lagoons. In the activated sludge field, 6 plants are equipped with lagoons.

Sludge Friction; Activated Sludge

For test purposes, some 8,200 ft of the 17-mile, 14-in. pipe line at Chicago was cleaned in 1942 with a go-devil. The coefficient *C* in the Hazen-Williams formula was increased from 85 to 124 which is 88% of the coefficient when the line was new.

Elutriation

At the end of 1943, there were 12 plants using the elutriation process in the United States. Of these three were built since 1941: Cranston, R. I., digested primary and waste activated sludge; and San Diego, Calif., and Stamford, Conn., digested primary sludge. Less complicated sludge piping and the quicker processing of the conditioned sludge are the novel features of these installations. At Springfield, Mass., sulfate of alumina has proved a reliable substitute for ferric chloride when coagulating well-elutriated sludge. The installation at Washington, D. C., still is the largest.

At the Hartford, Conn., plant, handling 25 mgd, 2 years of operation show (152) a low consumption of ferric chloride for sludge conditioning averaging 2.51%, estimated to be four tenths the amount required for unelutriated sludge based on the Baltimore tests of Keefer and Kratz (153). In 1941, $1,048 were saved and the digestion period reduced from 60 to 30 days. The saving in digestion tanks was greater than the cost of the elutriating tanks and equipment by $4,800. The use of ferric chloride was as low as 1.2%, with filter yields of 9 to 10 lb per sq ft per hr of cake with a moisture content of 60%.

At Annapolis, Md., on a plant serving 25,000 population, Weber reports (154) that elutriation saved in chemicals $5.94 per ton of dry solids, lengthened the life of filter cloths from 150 to 1,000 hours, and eliminated the need of acid washing of the cloths. One chemical, ferric sulfate, is used at a cost of $0.96 per ton of dry solids. The filter yield is now about 7.5 lb per sq ft per hr of dry solids. The filter is operated 145 days a year, and about 5 hours daily.

Pre-Aeration

Pre-aeration is suggested (155) as a preparatory treatment—not only to remove free gases from the sewage but also to promote flotation of grease and to some extent aid coagulation.

At Peoria (156) plant scale tests have shown that adequate pre-aeration of the sewage before its addition to the activated sludge results in improved operation of the activated sludge process. At Camp Edwards, Mass., plain aeration for 5 to 7 min in skimming-aeration tanks (after the Imhoff design) effects separation of the grease, although much of the removal is accomplished in the settling tanks which follow.

However, some authorities (20) point out that pre-aeration of the sewage at Boston was not successful in the over-all removal of grease and that the use of air alone, before sedimentation, in connection with Boston sewage would not be justified. Weston (21) goes further and states that the additional removal of fats by aeration does not appear to warrant the addition of aeration to plain sedimentation of the sewage without further treatment for the removal of grease. Apparently more data are required from actual scale tests.

According to Eliassen (157) a study of three types of equipment employed in army plants indicated that pre-aeration increased the removal of grease but slightly over sedimentation. The addition of chlorine (generally 5 ppm) to the aeration step or to aeration prior to vacuum flotation was of no apparent benefit. Gehm (158) made plant scale tests at South River, N. J., in an aeration tank with a detention period of 30 min, using 0.02 cu ft of air per gallon of sewage and 8 ppm of chlorine. The aeration tests lasted 8 hours for 30 days and the aero-chlorination tests 8 hours for 40 days. The pounds of grease removed in an 8-hr period were:

Description	Plain aeration	Aero-chlorination
Number of 8-hr periods	30	40
Pounds Removed per Million Gallons:		
Average	39	55
Maximum	85	97
Minimum	21	18

In this test the addition of chlorine to the air increased the removal from 39 to 55 lb per million gal or 41%.

Removal of Scum from Settling Tanks

The amount of floating scum in settling tanks varies greatly. In specially allocated skimming tanks mechanical devices are frequently installed, in connection either with rotary clarifier mechanisms or with the return of flights on the straight-line equipment. In certain situations where no mechanical devices are used, as in Imhoff tanks, the surface of the settling chamber is swept by pulling a floating timber through. In some cases water jets have been used (Fitchburg, Mass.). In South Africa at Johannesburg (159), the scum is swept into a radial scum pipe by successive releases of air from 6 radial perforated pipes installed below water level.

Mechanical Flocculation

The clarification of sewage can be improved by pre-flocculation without the addition of chemicals under favorable conditions. From the U. S. Public Health Service Census, the plants equipped for flocculation are allocated as follows:

Type of flocculation	Number
Mechanical	102
Air	15
Combined mechanical and air	8
Baffled mixing chamber	1
Type not stated	49

Of the total of 175 plants so equipped, at least 9 can be identified as using flocculation without chemicals. Fischer and Hillman (160) recommend that the detention time on raw sewage be 30 to 40 min in the flocculator and 1.5 to 2.0 hours in the clarifier and that the flocculator velocity should not exceed 1.5 ft per sec, with a maximum overflow rate of 800 gal per sq ft per 24 hr. For

trickling filter effluent treatment, 30-min flocculation and 1.0-hr settling at a 1,000 gal per sq ft per 24-hr overflow rate should be satisfactory.

At the West Side Works, Chicago, in 1937–1938, two tests, for flows of 2.99 mgd and 1.88 mgd, were run on pre-flocculation without chemicals, in comparison with plain sedimentation in Imhoff tanks. The effect was to increase the removal of suspended solids 6.5% and 5.6% and that of B.O.D. 12.3% and 0.9%, respectively, over plain settling.

A summary of the results is as follows:

Description	2.99 mgd	1.88 mgd
Operating Data:		
Mixing period (min)	4.3	6.8
Flocculation period (min)	28.2	22.0
Settling period (hr)	0.95	1.51
Chemical Data:		
Suspended Solids—		
Raw sewage (ppm)	123	127
Imhoff effluent (ppm)	70	71
Reduction by flocculation settling (%)	43.1	44.1
Reduction by plain settling (%)	36.6	38.5
Five-Day Biochemical Oxygen Demand—		
Raw sewage (ppm)	118	110
Imhoff effluent	71	67
Reduction by flocculation settling (%)	40.0	39.1
Reduction by plain settling	27.7	38.2

In 1939 at Minneapolis-St. Paul, experiments were conducted (161) for 8 days on half the plant using flocculation and settling as compared with settling alone in the other half (see Table 19).

TABLE 19.— MECHANICAL FLOCCULATION AND SEDIMENTATION; MINNEAPOLIS-ST. PAUL

Treatment	Detention period (hr)	Percentage Removal	
		B.O.D.	Suspended solids
Sedimentation only	1.64	44.2	75.5
Sedimentation plus flocculation	1.70	49.8	80.1

In the Circleville, Ohio, chemical precipitation plant (162) flocculation was tried with and without chemicals. On a design flow of 1 mgd, the flocculating tanks have a detention period of 32 min and the settling tanks, a detention period of 2 hr 20 min. The average daily flow is about 0.5 mgd in the winter and spring.

With flocculation, without chemicals, for 48 min, the suspended solids removed were as listed in Table 20. The use of chemicals increased the removal of suspended solids and B.O.D., respectively, to between 85% and 89% and between 77.4% and 79.4%, when using chlorinated copperas and lime or ferric chloride.

The results at Duluth, Minn., for the first 10 months in 1942 are shown in Table 21.

Vacuum Flotation

The so-called Vacuator system (163)(164) embodying vacuum flotation of suspended solids has three parts, the aerator, the de-aerator, and the vacuator. Pre-aeration is recommended with a use of air from 0.025 to 0.05 cu ft per gal of liquid treated for a period as low as 30 sec; de-aeration is recommended in a small compartment, followed by the exposure to a vacuum of about 9 in. of mercury in a special chamber equipped with a surface skimmer and a bottom raking mechanism. At present the principal tests available are on a pilot plant (7-ft-diameter vacuator) at the Los Angeles Terminal Island plant and on oil waste at a West Coast refinery. At Terminal Island, detention periods in the vacuator ranged from 2.8 to 20.8 min. The suspended solids varied from 357 to 544 ppm. The removal varied with the amount of pre-aeration and the detention period. With mechanical aeration removals from 36.6% to 51.3% were obtained. On oil refinery wastes, the best results were obtained by an added coagulant (usually caustic soda). To date 27 vacuator units have been installed, 3 for municipalities, 22 for industries, and 2 for cantonments. The largest municipal installation is at San Diego (average flow 20.6 mgd). As yet few operating data are available.

TABLE 20.—Flocculation Without Chemicals; Circleville, Ohio

Period 1940	Raw sewage (ppm)	Settled (ppm)	Removal (%)
January	414	89	78.5
February	533	159	70.1
March	468	164	64.9
April	305	98	67.8
May	419	84	79.9
Average	428	119	72.2

TABLE 21.—Use of Chemicals at Duluth, Minn., January through October, 1942

Determination	Average, 10 months	Monthly Average	
		Maximum	Minimum
Sewage flow (mgd)	11.8	14.8	8.3
Detention Period (hr):			
Flocculator	0.46	0.60	0.35
Clarifier	2.3	3.2	1.8
Five-day BOD:			
Raw sewage (ppm)	211.2	277.9	153.6
Final effluent (ppm)	132.4	172.8	108.8
Reduction (%)	37.1	39.9	33.1
Suspended Solids:			
Raw sewage (ppm)	161	206	121
Flocculator effluent (ppm)	171	212	131
Final effluent (ppm)	51	71	35
Reduction (%)	68.2	76.0	61.0

Chlorination

During the last 2 years, although the allocation of chlorine has been necessary, its application for sewage and water treatment was accorded the highest priority rating of any civilian use. This has always provided an adequate supply for both war needs and essential purposes (165). Under wartime condi-

tions, necessitating operation with existing equipment, no new large installations of chlorinating equipment have been made. According to the U. S. Public Health Service Census, at the end of 1941 there were 1,055 municipal sewage works equipped with chlorination apparatus and 163 institutional works, a total of 1,218.

At Buffalo in the fiscal years 1940–1941, the chlorine demand of the raw sewage was reported (166) as highly variable, ranging from 3.6 to 13.2 ppm. The expanded industrial activity in wartime has caused an average increase in chlorine demand to 6.27 ppm from 5.4 ppm in 1939–1940. During the last half of 1941 the chlorine demand in pounds was practically constant at 28% higher than in 1939–1940. However, the average dosage (6,610 lb per day) increased only 18.4% because extensive laboratory studies resulted in improved plant operation (166). The chlorine demand of the effluent was less than 1,000 lb per day. Chlorination of effluent at the Cleveland Easterly plant to maintain a residual of 0.5 to 1.5 ppm made a great improvement in the bacterial quality of seven adjacent bathing beaches. Not only is the effluent of the activated sludge plant much cleaner than the shore water of the lake, but, because of the chlorine residual, has a pronounced sterilizing effect on the lake water at the adjacent beaches. The bacterial pollution of the bathing beaches was reduced 91% (167). Toronto, Ont., Canada, awarded a contract in September, 1943, for the first unit of a sewage treatment plant, which will provide sedimentation, sludge digestion, and chlorination for a contributing population of 870,000 (168).

The use of chlorine at activated sludge plants to control bulking was reviewed (169) by a committee of the American Public Health Association in a report citing both favorable and unfavorable experiences. Definite advantages are shown to result from this application at 4 plants, while 2 other plants find no benefits (169). Two smaller plants have successfully used this control method for a considerable period of years (170)(171).

A survey of present research projects indicates that the use of chlorine for treatment of sewage and industrial wastes is receiving relatively little study (6). In a critical review of chlorine applications (172), however, Faber presents some new developments and discusses the opportunity for profitable research in this field. In addition to its recognized use in disinfection, chlorine appears peculiarly suited for controlling odors and retarding protein decomposition, and for coagulation and oxidation, either alone or in combination with other chemicals (172).

The treatment of industrial wastes presents many new problems, in which chlorination may be helpful. Plants are now being constructed to employ a chlorine process developed to recover grease from wool scouring wastes. Chlorine is used to destroy the toxicity of cyanide plating wastes and to counteract the reducing effect of sulfur dye wastes (172)(173).

A committee of the American Water Works Association has published a comprehensive report on the control of chlorination which, with necessary modifications, is proposed for adoption in the next edition of *Standard Methods* (174). Design details for chlorine control and storage rooms, for piping, and for safety precautions (175) have been indicated by Coburn.

The liberal chlorination of the final effluents of sewage treatment works at army camps is cited as being somewhat costly in operation but as economical, since it permits satisfactory operation in a war emergency with the provision of less biological treatment than might be required for permanent installations in municipalities (63). However, at many Army plants where biological treatment is provided, chlorination is not required.

Effect of Copper on Biological Processes

Moore and Kellerman (176) in 1904 indicated the value of copper sulfate in eradicating algae and other micro-organisms from reservoirs. In 1905 Johnson and Copeland (177) used copper sulfate at Columbus, Ohio, as a bactericide. In 24 hours contact amounts as low as 10 ppm gave a reduction of more than 99.99% in bacteria in trickling filter effluent and in raw sewage. In 1907 Kellerman, Pratt, and Kimberly (178) noted the high germicidal value of copper sulfate when acting on partly purified sewage. At New Haven, Conn., in 1918 Winslow and Mohlman (179) found copper (in the form of copper sulfate) in the East Street sewage, ranging from 1.9 to 8.8 ppm. This came from spent acid used in pickling brass shells. When 5.6 to 8.8 ppm were present, the bacterial content of the sewage dropped from 990,000 to 3,000 (20° C) per cu cm. Winslow and Mohlman concluded that

> "the operation of Imhoff tanks or of the activated sludge process, or of any other method of sewage treatment which depends on biological activity, would be seriously interfered with, in the case of the East Street sewer, by the presence of antiseptic industrial wastes."

Barry states that in 1937 when constructing the new Boulevard System at New Haven, dewatering and incineration of sludge was chosen to avoid possible trouble with large amounts of copper in the sewage.

Recently trouble has occurred (180) at Kenosha, Wis., in the digestion of sludge deposited from sewage containing waste from brass factories. The city has a population of 49,000 and a sewage flow of 9 mgd, including varied industrial wastes. The sewage treatment plant has primary treatment with separate sludge digestion. Poor digestion was traced to rinse waters (1.7 million gal discharged in 10 hr) from a brass plant carrying from 10 to 130 ppm of copper. The bottom sludge contained 3,000 ppm of copper. Laboratory tests showed that a copper content of 200 ppm in sludge inhibited digestion.

Metal plating solutions can be handled at sewage works, even in sludge digesters, if the copper concentration does not exceed 10 ppm. In one city filtration of raw sludge is necessary because the high copper concentration upsets digestion. Hoover (181) states that when copper occurs in sewage in an amount over 1 ppm the efficiency of sludge digestion is lowered.

Chromium wastes do not accumulate in sewage sludge to the same degree as copper wastes, but, if precipitated, they show comparable harmful effects. If more than 200 ppm of precipitated chromium are present in sludge, the rate of digestion is markedly reduced. When more than 1 ppm of chromium is present in sewage arriving at a plant using sludge digestion, the chromium should be removed at the source. Chromates also affect biological action.

Jenkins and Hewitt state (182) that

> "from 5 to 10 ppm of chromium in form of CrO_4 in the sewage arrested the production of nitrate and nitrite from NH_3 by activated sludge within 48 hours. The inhibitory action lasted for a week after the removal of this inhibiting agent."

Spent Pickling Liquors

Hodge (183) has reviewed the processes devised for the recovery of iron salts from spent acid pickling liquors. Such processes usually separate ferrous sulfate from the liquor by crystallization brought about by evaporation and cooling. Even the best of these processes has not proved attractive. Gehm has discussed (184) a process patented by de Lattre (185) which employs methanol to produce crystallization. The alcohol is recovered by distillation. The recovered acid is made up to strength and returned to the pickling vats. The crystalline copperas is dried. Gehm believes acetone is superior to methanol. Apparently processes which recover ferrous sulfate are not attractive to industry, because that by-product is a drug on the market. Until some new outlets can be found or some different products, the treatment of waste pickling liquors is not satisfactorily solved, from the standpoint of recovery of by-products.

Some years ago at Chicago the industry arranged to remove practically all the waste pickling liquor from the sewers for direct discharge to the river and channel system. Recently Hammond, Ind., and Crawfordsville, Ind., have been added to the growing list of sewage plants which have by-passed spent pickling liquor wastes direct from the mill to the stream.

Chemical Treatment

The enthusiasm of earlier years for chemical precipitation has died down. Apparently only one chemical plant was built in 1941. The total number of plants as of the end of 1941 was 186, according to the U. S. Public Health Service Census.

The seasonal use of chemicals is established at the Shades Valley plant at Birmingham, Ala., Coney Island, N. Y., and Waukegan, Ill. Equipment is being provided at the Owls Head (New York City) plant under design. At Waukegan, the operating experience during six summers indicates that ferric chloride or sulfate of alumina with sodium silicate are the best. At the Atlanta (Ga.) Clayton plant the equipment was operated in an experimental way in the falls of 1941 and 1942, using ferric sulfate intermittently to increase the performance of settling tanks. At Minneapolis-St. Paul in the first 4½ years of operation the chemical equipment was operated in one half the plant for 4 days to make an acceptance test. Using 12 ppm of ferric sulfate the respective reductions in the settling tanks were:

Treatment	B.O.D. removed (%)	Suspended solids removed (%)
Sedimentation (1 hr)	35.5	73.0
Sedimentation plus chemical (30-min flocculation with air, plus 1-hr settling	55.5	83.4

Along the Atlantic seaboard no significant change is observed in the chemical treatment of sewage where used. Most of the smaller New Jersey plants, such as Lakewood or Essex Fells, continue to employ sulfate of alumina. In New York, Freeport, L. I., has changed from ferric chloride to ferrisul as a matter of convenience, but Coney Island still uses chlorinated copperas, chiefly on account of the chemical handling problem. In New Jersey, where intermittent sand filters are prevalent, there is an advantage in the use of chemical precipitation because the State Board of Health regulations permit a loading of 400,000 gal per acre per day with chemical precipitation effluent as compared with a loading of 150,000 gal per acre per day following plain sedimentation.

Guggenheim Process

The Guggenheim (so-called "Biochemical") process has not yet reached a stable position in the field of sewage treatment. In 1943, there were 5 municipal plants in operation and 3 industrial plants. From the laboratory standpoint the study of Phelps and Bevan (186) is of interest, although on a very small scale. Two pilot plants handled 120 gal per day each for 10½ months, using synthetic sewage. Ferric sulfate was applied at the rate of 5 ppm. The application of iron in an amount from 5 to 10 ppm is claimed to stimulate the growth of certain organisms and, after conditioning, to increase the rate of oxidation of organic matter. The so-called "biochemical" sludge is claimed to adsorb more rapidly, accompanied by the coagulating action of the applied ferric sulfate thus reducing the time of aeration and the amount of air. In the tests the aeration period was 3 hours for the Guggenheim process and 6 hours for the activated sludge process. However, Gehm (186) questions the value of the pilot plant tests with activated sludge because of the low solid content (630 ppm) in the mixed liquor.

This Committee is doubtful how far tests on such a small scale can be relied upon.

The Hays Process

The Hays process of sewage treatment (187) is a type of contact-aeration plant which has been used in Army and Navy installations largely in the Southern States. Originally a few small municipal plants were built in Texas, the first in 1938 at Elgin. Up to 1941, when the process was first used by the Army, 5 or 6 more municipal plants were either under construction or in operation. The first plants built by the Army were small. The initial operation experience was claimed to be favorable both in the Army and in the small municipal installations. The intense military construction program of 1941 and 1942 included some 70 Hays plants, designed to serve populations ranging from 1,000 to 35,000. Because of lack of precedent, the design of these plants was based on the early installations. As a result, mechanical, structural, and operational difficulties developed rapidly and reached a point in July, 1942, where the construction of plants of this type was discontinued by the Army. The operation of the plants as a whole has been difficult. Even with high rates of use of air, there is trouble in handling strong sewage.

Operational data are retained by the Army and Navy. The published information is unfavorable both as regards efficiency and economy. Recent

experience in the Army installations indicates that in some cases an acceptable effluent can be produced by relatively minor plant revisions and a more general understanding of the particular plant requirements. However, the costs of construction, maintenance, and operation now appear to be relatively high in existing installations, although there is some indication that these costs may be lowered. Kessler and Norgaard (71) state that "perhaps the Hays process plant can continuously produce a highly purified effluent but when such is the case it will be accomplished at a cost that is relatively high and the needs in most Army camps where the plants are now in operation could be met more economically by other methods."

Comments on the Hays process from various viewpoints are available from Griffith (188), Greeley (123), and Hurley (189). Judgment of the merits of this process will depend largely on data now withheld. Contact-aeration, as used in the Hays process, with adequate details may be capable of producing a high degree of purification, but little is known about the relation between loading and cost. Thus, the ultimate status of the process is questionable. Present indications are that the first cost is too high for the treatment obtained (123).

Mallory Process

Mallory (190)(191)(192)(193) describes a procedure for controlling the treatment of sewage by aeration, in which sludge is used derived from previous aerations. Apparently this treatment is covered by the common or usual definition of the activated sludge process. Mallory advances a theory that the activated sludge process follows certain definite mathematical formulas and is capable of absolute control both in the design of the plant and in its operation (194). He claims the process does not depend on biological activity. For control he relies on the determination of the sewage flow, a centrifuge test for the solids, the measurement of the depth of the sludge blanket, and the determination of the volume of the sludge after settling for 1 hour in a wide 2-liter graduate (expressed as cubic centimeters per liter). In his writings he apparently ignores the strength of the sewage, the proportion of volatile matter in the suspended solids, the air requirements, and the dissolved oxygen to be maintained.

Mallory claims that among the principal controlling factors in the operation of the activated sludge process are (1) the relationship between the effective period of aeration and of sedimentation, (2) the relation between the concentration of suspended solids in the aeration tank and in the return sludge, (3) the distribution of the suspended solids in the system between the aerator and the settling tank representing a definite relationship to the aerator-sedimentation ratio, and (4) the ratio of the capacity of the secondary settling tank to the volume of sludge settled in such a tank (sludge blanket). These factors are combined in various indexes. One is styled an "equilibrium index," which has an optimum value of 100. Values greater than 100 indicate a safety factor and those less than 100 indicate substandard conditions. Another is styled the "oxidation index." To determine the value of his suggestions, further and more complete data are required from actual installations presented by independent observers.

Combined Garbage and Sewage Disposal

Combined sewage and garbage disposal plants have been built at Findlay, Ohio; Lansing and Midland, Mich.; Goshen, N. Y.; Gary and Marion, Ind.; and Rock Island, Ill. At the first 5 plants dual disposal is used. At Lansing and Goshen the garbage is added directly to the digester, whereas at Findlay and Marion garbage is added to the sewage ahead of the plant. Formerly, at Midland, the garbage was added to the digester, but, in 1942 and since, it has been added to the sewage. At Rock Island and Gary, no garbage has been handled. At Grand Rapids, Mich., the garbage is mostly fed to hogs. Any excess is ground and discharged directly to the digesters.

For Lansing (195) from July, 1940, to June, 1941, inclusive, with 75,000 people, the average results are given in Table 22.

TABLE 22.—Combined Garbage and Sewage Disposal; Lansing, Mich.

Description	Pounds per day	Percentages	
		Solid	Volatile
Raw primary activated sludge	619,000	4.69	58.5
Garbage	47,600	19.1	91.6
Sludge and Garbage:			
Mixture	666,600	5.71	66.5
Digested	116,500	9.30	47.7

Digestion proceeds rapidly with a gas production of 3 to 3.5 cu ft per capita or 9.5 cu ft per lb of volatile solids put into the digestion tank. Tests indicate that stage digestion is desirable. With 4 tanks used in series, 1.7 cu ft per capita may suffice, compared with 4 cu ft per capita for single-stage digestion. Filterable sludge was obtained in 15 to 16 days. Operation developed some trouble, principally due to bones, egg shells, glass, etc., which obstructed piping and clogged valve bonnets, fittings, and pumps. Plug valves proved preferable to disk valves. According to Wyllie clogging of pipes still occurs, although a velocity of 2.5 ft per sec minimizes the difficulty. From the digesters, now greatly overloaded, the supernatant liquor removed is very high in suspended solids (around 6%).

Tolman (196) discusses dual disposal at Findlay (35,000 population) and Marion (40,000 population). Both plants are of the activated sludge type, with separate digestion of sludge. About half the garbage is ground once or twice daily at the plant, the remainder being fed to hogs. In 1941–1942, the garbage averaged 0.25 lb per capita per day at Findlay and 0.23 at Marion. Tolman concludes that, when ground garbage is added to raw sewage at the rate of one ton per million gallons, the suspended solids and B.O.D. are increased less than 10%; the combined solids settle more efficiently; the garbage does not affect adversely the oxidizing ability of activated sludge; and the combined solids digest satisfactorily to a lower solids content than those from sewage alone. He recommends that all grit, bones, glass, etc., be removed from the garbage; two-stage digestion be provided, with 4 cu ft per capita for primary solids and garbage, and at least 7 cu ft per capita for primary solids, garbage, and activ-

ated sludge. For the years 1941–1942, the ratio of suspended solids arriving at the plant to the garbage solids averaged 4.26 for Findlay and 23.3 for Marion. The gas produced from the digestion of the garbage supplied the power to compress the air for oxidizing the garbage B.O.D. and left a surplus.

At Midland (197) since January 1, 1942, the garbage (about 1¼ tons per day) of some 10,000 people has been handled with the sewage in a plant comprising a comminutor, detritor, flocculators, and clarifiers. The sludge is digested with heat control, conditioned with about 2.5% ferric chloride, and dewatered on a vacuum filter to a cake of 55% moisture, containing about 70% ash. This is high, because of the presence of certain mineral wastes from chemical works. Such sludge is readily pumped with 30% solids. The garbage is ground, mixed with water, and discharged into a wet well, which traps troublesome solids. The garbage increased the gas output about 0.27 cu ft per capita per day. Prior thereto, the gas output was between 0.40 and 1 cu ft per capita per day.

The annual averages at Midland, before and after the addition of garbage, were as follows:

Data	1941	1942
Sewage pumped (million gal)	511	522
Raw Sewage Analysis:		
Suspended solids (ppm)	169	208
Removal (%)	59	61
Five-day B.O.D. (ppm)	134	176
Removal (%)	29	34
Gas production (thousand cu ft)	3,178	4,108
Digested Sludge:		
Total solids (%)	29.4	21
Volatile (%)	29.67	32
Dried sludge after filtration (cu yd)	608	780
Garbage added (tons)	0	413

Utilization of Sludge Gas

The utilization of digester gas in engines for power (or air compression) and waste heat recovery continues to grow. In 1941 (99) there were 133 installations in the United States. In New York City, the success at the Coney Island plant was followed up in the Tallmans Island, Bowery Bay, and Jamaica plants. These 4 plants are now in operation with a total of 11,500 bhp. The 26th Ward, Owls Head, Hunts Point, and Rockaway plants, now being designed, include engines totaling 18,300 bhp. This total horsepower (nearly 30,000 bhp) in New York City is a respectable proportion of the gas engines installed in the United States operating on digester gas. The gas engine units in New York City vary from the original 300 hp to 1,450 hp. Washington, D.C., has a 1,200-hp unit. However, most of the engines in the United States are under 500 hp (198)(199).

In relation to the power supplied from producer gas, at the respective plants, Coney Island is self-sufficient. Tallmans Island is substantially self-sufficient for sewage pumping and blowers. Bowery Bay, with a relatively low gas pro-

duction, supplies only up to one half of its requirements. Jamaica (also an activated sludge plant) has not operated long enough to permit conclusions.

In some installations the sludge is prepared for use in engines by scrubbing (200) and also by passing through iron sponge filters to remove hydrogen sulfide (201). At Aurora (202) a greater content of hydrogen sulfide in digester gas has increased corrosion. In 1941 the sulfur content was double that of 1940, and three times that of 1939.

The use of compressed sludge gas in steel bottles for motor truck fuel has been tried at the Georgia School of Technology (203) in Atlanta and by the City of Atlanta (204). This scheme was used in Germany many years ago, according to Imhoff (205) and Heilmann (206).

Use of Sewage Sludge as Fertilizer

Under war conditions interest has increased in the use of air-dried digested sludge as a fertilizer and soil conditioner (207). Gary, alone, reports a decreased interest (208). A few cities sell heat-dried digested sludge. Dayton, Ohio, reports sales in bulk carload lots at $7.50 per ton. Springfield, Mass., receives $5.00 per ton, f.o.b. containers furnished by the customer. Many smaller cities give air-dried digested sludge to city parks and farmers, free or with a nominal charge for loading. In some cases the sludge is removed from the drying beds and loaded by the taker. Apparently where organic fertilizer is relatively scarce, as in California, higher prices are realized for air-dried digested sludge than in areas east of the Mississippi River. Fresno, Calif., reports (209) sales of air-dried digested sludge in 1943 for $4.00 per ton f.o.b. the drying beds, containing 2.11% nitrogen on a dry basis. Spartanburg, S. C. (210), has sold on a basis of $1.00 per ton on a dry basis plus $1.00 per ton for each unit of nitrogen. The use of a suitable disintegrator for pulverizing air-dried sludge is increasing.

Rudolfs and Gehm (211) discuss the chemical composition of sewage sludges and the content of growth-promoting substances. They conclude that "the value of sludge as a subgrade fertilizer or soil builder cannot be determined by chemical analyses alone, but must be judged by the results obtained. Sludges are not balanced plant foods but contain major and minor mineral elements, growth-promoting substances, and varying amounts of organic matter." As a substitute for well-rotted manure, a disintegrated air-dried digested sludge is of value. Kraus (212) indicates that sewage contains a growth-promoting vitamin with properties similar to those of riboflavin.

The literature on the minor elements and their relation to plant and animal nutrition is brought up to date by Willis (213) in a collection of abstracts. Recently Bear (214) has emphasized the need of boron, magnesium, and calcium in certain soils, in addition to the usual application of nitrogen, phosphoric acid, and potash. The liberal use of finely ground dolomitic limestone may be helpful.

The conclusions of the Committee on Sewage Disposal of the American Public Health Association (215) in its report on the utilization of sewage sludge for fertilizer are still sound. A few new names for commercial brands of sludge sold as fertilizer have been added:

Schnectady, N. Y. .... Gro-humous
Clearwater, Fla. .... Clear-O-Sludge
Oshkosh, Wis. .... Oshkonite (216)

At Appleton, Wis. (217), a digested sludge is fortified by adding chemicals to make a mixture containing 5% nitrogen, 2.1% P_2O_5, and 2.1% potash. This is sold as "Appcolizer."

Rudolfs and Gehm (211) find the average phosphoric acid content of sewage sludge varies from about 1.50% to 4.60%. Digested sludge contains more phosphoric acid than plain settled sludge, and digested chemically-precipitated sludge has the highest phosphoric acid content. Chemical treatment does not increase the insoluble phosphoric acid.

Heat-dried activated sludge at present is produced by only four plants—Chicago (Calumet and Southwest), Houston, and Milwaukee. Pasadena, Calif., has ceased production and now discharges its excess sludge into the trunk sewers of the Los Angeles County Sanitation Districts for disposal in the Pacific Ocean. However, much of the sludge is recovered in the treatment works of the districts, digested, and air-dried for sale at $2.00 per ton on the drying beds. The purchaser distributes this material to orange groves at $7.00 per ton. Milwaukee still sells about half its output, bagged, to jobbers. The remainder goes into the wholesale market in bulk in carload lots to mixers, as does the product of Chicago and Houston. Such sales are based on analysis. As of September 17, 1943, the Office of Price Administration (OPA) has set the maximum price f.o.b. the plant per unit of ammonia at $3.00 for Milwaukee, $2.80 for Chicago, and $2.60 for Houston, plus $0.40 per unit of available phosphoric acid.

The outlook for organic nitrogenous material after the war is uncertain. Any city contemplating the manufacture of heat-dried activated sludge should carefully investigate costs and returns, and in any case provide for other ways of disposal, such as incineration of dried sludge. Under war conditions, the installation of heat-drying equipment is banned.

Fertilizer in Wartime

Brand (218) states that the consumption of fertilizer in the United States increased from 7,571,000 tons in 1938 to slightly more than 10,000,000 tons in 1942 and will probably top 10,500,000 in 1943. In the allocation of chemical nitrogen, ammunition came first. The use of chemical nitrogen was prohibited on fall-sown small grains, on certain nonessential crops, on golf courses, lawns, and on noncommercial ornamental plantings. In 1942 approximately 415,000 tons of nitrogen were used by farmers but this supply did not meet their demand. In 1941 456,000 tons were used by farmers. The 10,000,000 tons of mixed fertilizers and fertilizer materials used on crops in 1942 may be compared with 7,571,000 tons in 1938 and 9,240,000 tons in 1941.

The shortage of chemical nitrogen in 1942–1943 caused a vigorous effort to increase the supply. In normal times organic nitrogen carriers would be substituted but all such suitable materials were diverted to feed. Record amounts of by-product sulfate of ammonia were produced. More than 1,000,000 tons of

nitrate of soda were imported from Chile in the 12-month period ending June 30, 1943. In the summer of 1943, restrictions were somewhat eased.

The supply of organic nitrogen is not entirely adequate. Because of the demand for animal feed, the use of edible oil-seed meals (cotton seed, soybean, and peanut) is prohibited in fertilizer. Approximately only 60% of the organic nitrogen used in fertilizers in the 1941–1942 season will be obtainable in the 1943–1944 season, which probably will not meet the needs of fertilizer manufacturers.

Effect of Sludge Use on Health

Wright, Cram, and Nolan (219) found animal intestinal parasites in sewage and sewage sludge from a number of army camps. *Helminth* ova were present in 55% of the samples of sludge from secondary digesters. *Trichuris* eggs, recovered from primary and secondary digesters, appeared to retain their viability. Cysts resembling those of *Endamoeba histolytica* were found in sludge from the bottom of a final digestion tank in California. Their findings indicate the possibility that fresh or digested sludge used as fertilizer may disseminate the ova of intestinal parasites. Further experiments by Cram (220) under controlled conditions indicate that cysts of *Endamoeba histolytica*, hookworm, and *Ascaris* eggs remain in the effluent of settling tanks. *Amoeba* cysts and *helminth* ova may pass through trickling filters or activated sludge. However, chemical precipitation or intermittent sand filtration removed *amoeba* cysts. Apparently *Endamoeba histolytica* did not withstand digestion, but *Ascaris* and hookworm ova survived digestion and subsequent air drying of sludge. Heating of dried sludge to 103° C for 3 min destroyed all the *Ascaris* eggs. Cram believes that the cysts and ova of the intestinal parasites will not survive an exposure to a temperature of 700° to 800° F for 5 min or longer. *Ascaris* eggs are considered more resistant than hookworm eggs or larvae or *Endamoeba histolytica* cysts. On this basis heat drying of sludge is effective. The careless use of air-dried sludge or wet sludge as fertilizer may be hazardous from a public health viewpoint. Hence such sludge should be treated like manure. Many phases of the problem still need investigation.

Cram states that there is no danger of the spread of human intestinal parasites through the use of horse or cow manure. On the other hand, there is danger of the spread of parasites to horses or cows from use of their own manure, especially when fresh, just as there is danger to man from the use of sludge containing human excreta.

In the sewage of Moscow, Russia, Gordon (221) found certain viable cysts (*Endamoeba histolytica;* E. coli; *Giardia intestinalis; Endolimax nana;* and *Iodamoeba bütschlii*) which were not eliminated by settling (up to 6½ hours) or coagulation by ferric chloride. Filtration through 2 m of soil removed them all. Apparently chlorine up to 12.7 mg per liter of residual chlorine, acting for 30 min, had no effect.

Odor Control

On account of the limited isolation of the sewage treatment works in New York City, odor control is important. At Wards Island, activated carbon and ozone were installed in 1938 to control odors from stale sludge stored at the

dock awaiting disposal at sea. Activated carbon was also installed on the sludge vessels for the control of odors en route to the dumping grounds. Experience indicates that activated carbon is effective in reducing odors, but rather expensive, whereas ozone may be cheaper and equally effective. The Owls Head plant under design includes ozone control. However, a simpler solution is being sought experimentally.

Rural Sanitation

For those concerned with rural sanitation, the recommendations of the Joint Committee on Rural Sanitation and Rural Sewage Disposal, a group selected from various governmental and state agencies (222), outline conservative practice for small installations, both with and without water carriage.

Patents in Sewage Treatment

From 1939 to 1942, an unusual number of sewage treatment works were built, in which considerable mechanical equipment was used. In the past 30 years numerous patents on apparatus and processes have been issued, which in many cases were and are offered for use without adequate experience or design data. The effect on the development of the art is open to question. Where a background of adequate test and experience is behind a patent, the disclosures contained therein may have aided the development of the art.

Attempts to interpret patents so as to cover devices, apparatus, or methods lacking in novelty, to influence contracts, are unethical.

The question has also been raised as to the extent to which an equipment manufacturer should furnish free design service in the hope that his product may be used. This practice has probably existed for over 40 years in small situations, and now seems to be growing, accelerated, perhaps, by the demand for sewage works under conditions of war stress and the tendency for engineers to look to contractors for help in design. Thus to an increasing degree the engineer is faced with the problem of designing works to be built with the widest competition and at the same time of securing what he considers the best practice for his client through a welter of claims by owners of patents.

Some engineers have asked why this Committee or some special committee could not pass on the value and validity of the patents in the sewage treatment field. Obviously such a procedure is impracticable, because from a legal point of view the validity can only be finally established by adjudication. The value, if any, is established by practice. Some patents have what lawyers call a "nuisance value," which means that they form a basis for a threat, either of suit for infringement or of injunction against a pending award of contract. Naturally, the more thoroughly engineers inform themselves about the history of the art and of patents (which are a form of publication) the less weight such nuisance patents can have.

The life of a patent by law is fixed at 17 years from the date of issue. In the history of the patent office few patents have been extended beyond that period. But the owner of a patent, by developing improvements and patenting them from time to time, may hold or increase his business position for a far longer period.

Prior art or publication may bar or invalidate the issuance of a patent. Consequently, if engineers will publish freely and accurately the essential things they do, both novel and routine, this may prevent the improper issuance of many patents. If an engineer desires to protect his invention by a patent, he should consult a lawyer familiar with patent practice in order to protect his rights.

In the examination of a patent a question which often arises is whether what is patented is merely what any engineer familiar with the art would do. This is particularly applicable in many patents concerned with civil engineering. The attitude of the courts in passing on patents may change from time to time. Only a lawyer thoroughly familiar with patent procedure and precedents is able to advise on patents and their interpretation and even then his judgment may be upset by the courts.

Among the journals in the field of sanitary engineering, *Water Works and Sewerage* and *Chemical Abstracts* list the new patents as issued by the United States. *Chemical Abstracts* also lists foreign patents. The Patent Office issues a biweekly journal, the *Patent Office Gazette*, in which all U. S. patents and trade-marks are recorded. Those who wish to be informed on the general principles of patent law may profit by reading books by Biesterfeld (223) and by Hoar (224). For advanced study, lawyers turn to Walker (225) whose treatise on patent law has run through many editions.

Infantile Paralysis

Infantile paralysis or poliomyelitis has been known to the medical world for more than 100 years according to Rosenau (226). Even today the theories on the host and transmission of the disease are still diverse and conflicting (227) largely because of the difficulty of intensive study. All authorities agree, however, that it is a "virus" disease, which will pass through the types of filter commonly used to remove bacteria. Poliomyelitis is also an organism which can only be cultivated on animal tissue. Many types of "polio" are recognized, of which the human type is one. For the study of the human "polio," certain monkeys are the only medium available. Mice or rats are unsuitable, as mice have their own peculiar type of "polio."

Present research indicates a reservoir of potential poliomyelitis. The virus was first obtained from fecal matter in 1912 by Kling, Petterson, and Wernstedt (228). Their investigations were corroborated by Sawyer (229), Harmon (230), and others (231)(232). Thus there is ample proof that it occurs in human feces, not only from those clearly afflicted with the disease, but from others apparently well. It is further found in the walls and secretions of the pharynx.

Rosenau (226) states the theories of transmission include contact with mouth or nose discharges and transmittal by insects, milk (from observation of certain cases using unpasteurized milk), food and drink, and dust (considered remote).

There is still doubt as to the portal of entry of the virus in human beings. Infection seldom occurs by way of the olfactory tract but mainly through the mucous membrane of the pharyngeal or the lower gastro-intestinal tract, or both. In the use of monkeys for test, the virus is usually injected in the brain or spinal cord. However, monkeys have been infected (226) by placing virus

material in the stomach. For 25 years or more the transfer was usually successful through the nasal passages of the monkey, though sometimes negative (232).

Paul, Trask, and Culotta (233) found poliomyelitis virus in sewage at Charleston, S. C., and Detroit, Mich. (234). However, Trask and Paul (234) (235) doubt if the disease is directly disseminated by water, in which Maxcy (236) concurs. Some believe the virus is more resistant than B. coli organisms, although tests in 1941 gave rather erratic results. In one instance (237) the virus was inactivated by 0.5 ppm of chlorine; in another, it survived as high as 15 ppm. Kempf, Pierce, and Soule (238) report that, using hypochlorite solution, the presence of 1 ppm of chlorine had no effect in 25 min, but that 1.5 ppm inactivated the virus in 20 min. Maxcy and Howe (239) state that "there is no justification for the statement that present day methods of chlorination are of little value in the destruction of the virus."

Recently Carlson, Ridenour, and McKhann (240) investigated the effect of activated sludge from a municipal sewage treatment plant on the removal or inactivation of a mouse-adapted strain of poliomyelitis virus. They found that activated sludge in amounts as low as 1,100 ppm, with 6 hours' aeration, removed or inactivated the virus to a sufficient extent to render it noneffective to mice when injected intracerebrally.

Maxcy and Howe (239) state that

> "* * * from the epidemiological point of view poliomyelitis does not behave like a waterborne disease. It has never been correlated with poor water supplies nor have explosive outbreaks of widely scattered cases appeared in cities with municipal water systems. Furthermore, cities with water sources located in remote spots far from human habitation suffer from poliomyelitis as frequently as those which obtain their water from sewage polluted streams."

Maxcy and Howe discuss the possibility that flies may play a part, and conclude that "their role in dissemination of the disease is as yet unknown." However, according to these writers, present knowledge does not resolve the question as to whether poliomyelitis is spread by fecal contamination of food, drink, objects, or hands, or by the transmission of droplets of pharyngeal mucous, or by both.

Thus the life history of poliomyelitis and its viability in sewage or water is still very vague. On the other hand, the sewage works operator should carefully watch all places where flies may congregate around the sewage works in contact with raw sewage or raw sewage materials. Until continued research has pulled back the veil and revealed the host and the methods of infection, sewers and sewage treatment works seem relatively unimportant as a source of information.

The present situation has been concisely summed up by the editor of the *American Journal of Public Health* (241):

> "All in all, the situation strongly suggests a leanness of interpretable facts and a fatness of hard-headed opinions. In such circumstances only the open-minded may claim epidemiologic respectability and they, unlike

Pontius Pilate, must tarry for an answer, unashamed to admit ignorance, unawed by authoritative edicts, persistent and diligent in the search of truth."

Respectfully submitted,

WELLINGTON DONALDSON	E. T. KILLAM
SAMUEL M. ELLSWORTH	WILLEM RUDOLFS
LINN H. ENSLOW	M. T. SINGLETON
C. E. KEEFER	F. M. VEATCH

LANGDON PEARSE, *Chairman*

Committee on Sewerage and Sewage Treatment

January 20, 1944

APPENDIX

BIBLIOGRAPHY OF REFERENCES

(1) **First Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment,** *Proceedings,* Am. Soc. C. E., November, 1942, Vol. 68, pp. 1559–1584.

(2) **637 Sewage Plants under Construction during 1941,** by M. M. Cohn, *Sewage Works Engineering,* February, 1942, Vol. 13, pp. 74–88.

(3) **The Influence of This War upon Sewerage Problems,** by A. M. Rawn, *Sewage Works J.*, November, 1942, Vol. 14, pp. 1217–1227.

(4) **Sewage Disposal Post-war Prospects,** *Engineering News-Record,* October 7, 1943, Vol. 131, pp. 544–545.

(5) **A Critical Review of the Literature on Sewage and Waste Treatment and Stream Pollution,** Committee on Research, Federation of Sewage Works Associations, *Sewage Works J.* (for Year 1941), March, 1942, Vol. 14, pp. 317–367; and (for Year 1942), March, 1943, Vol. 15, pp. 197–241.

(6) **Committee on Research Federation of Sewage Works Associations,** Survey of Sewage Research Projects (1943), *ibid.*, Vol. 15, May, 1943, pp. 466–476; and July, 1943, pp. 702–709.

(7) **Ohio River Pollution Survey, Final Report to the Ohio River Committee (1942):**

Vol. I. Main Ohio River and Minor Tributaries.
Vol. II. Major Tributaries.
Vol. III. Major Tributaries.

Supplement:

A. Collection of Data on Sources of Pollution.
B. Organization and Methods of Laboratory Studies.
C. Acid Mine Drainage Studies.
D. Industrial Waste Guides.
E. Epidemiological Studies.
F. Biological Studies.

(8) **Red River of the North Research Investigation,** a report prepared by the North Dakota State Dept. of Health, covering the Joint Investiga-

tion of the North Dakota State Dept. of Health, the Minnesota State Board of Health, in collaboration with the U. S. Public Health Service, 1938–1941.

(9) **Red River of the North,** Report on Regional Planning, Part V, National Resources Committee (1937).

(10) **Study of the Pollution of the Meramec River,** by W. Q. Kehr and W. S. Johnson, Report Engineering Division, Missouri State Board of Health (1943).

(11) **Investigation of the Pollution of the Androscoggin River and Remedial Measures,** by Metcalf and Eddy, Main Sanitary Water Board, *Bull. No. 1* (1942).

(12) **Raritan River Pollution Studies,** by W. Rudolfs and H. Heukelekian. Comparison of Results Obtained in 1927–1928; 1937–1938; 1940–1941, *Sewage Works J.*, July, 1942, Vol. 14, pp. 839–865.

(13) **Tennessee Valley Authority. Studies of the Pollution of the Tennessee River System,** I. General Description, by G. R. Scott (1941).

(14) **A Study of the Stream Pollution Problem in the Roanoke, Virginia, Metropolitan District,** by P. H. McGauhey, H. F. Eich, H. W. Jackson, and C. Henderson. *Bulletin*, Virginia Polytechnical Inst., Eng. Experiment Station, May, 1942.

(15) **Investigation of the Odor Nuisance Occurring in the Madison Lakes from July, 1942 to July, 1943,** Committee Report, D. W. Mead, et al. (1943).

(16) **The Working of the Madison Lakes,** by W. Trelease, Wisconsin Academy of Sciences, Arts and Letters, Vol. 7, pp. 121–128 (1883–1887).

(17) **Investigation of Odors from Lake at Madison, Wis.,** by J. W. Alvord, *Engineering News-Record,* September 4, 1919, Vol. 83, p. 469.

(18) **Treatment of Algae and Weeds in Lakes at Madison,** by B. P. Domogalla, *ibid.*, December 9, 1926, Vol. 97, pp. 950–954.

(19) **Report upon the Collection, Treatment and Disposal of Sewage and Industrial Wastes of the East Bay Cities, California,** Alameda, Albany, Berkeley, Emeryville, Oakland, Piedmont, Richmond, by C. G. Hyde, H. F. Gray, and A. M. Rawn (1941).

(20) **Report of the Special Commission on the Investigations of the Discharge of Sewage into Boston Harbor and Its Tributaries,** by A. D. Weston, Mass. House Doc. No. 1600 (1936).

(21) **The Grease Problem in Sewage Treatment,** Discussion by A. D. Weston, *Proceedings,* Am. Soc. C. E., February, 1942, Vol. 68, pp. 269–279.

(22) **Report on Sewage Disposal, Hampton Roads Sanitation Commission** (1942).

(23) **Interstate Sanitation Commission, Connecticut, New Jersey, and New York,** Annual Reports, Nos. 1 to 6, Years 1937 to 1942, inclusive.

(24) **Pollution of Everett Harbor,** by L. D. Townsend, A. Ericksen, and H. Cheyne, State Pollution Commission, Pollution Series *Bull. No. 3,* September, 1941.

(25) **An Analysis of Stream Pollution and Stream Standards,** by G. J. Schroepfer, *Sewage Works J.*, September, 1942, Vol. 14, pp. 1030–1063.

(26) **Classification of Inland and Shore Waters.** Sub-committee on Classification of Streams in New England, Central New England Drainage Basin Committee, National Resources Planning Board, by W. J. Scott et al., *Sewage Works J.*, pp. 1064–1073.

(27) **Disposal of Sewage into the Pacific Ocean,** by A. K. Warren and A. M. Rawn, *Modern Sewage Disposal*, pp. 202–208 (1938).

(28) **Predetermining the Extent of a Sewage Field in Sea Water,** by A. M. Rawn and H. K. Palmer, *Transactions*, Am. Soc. C. E., Vol. 94, pp. 1036–1081 (1930).

(29) **Pollution Survey of Santa Monica Bay Beaches in 1942,** Report by C. G. Gillespie, Bureau of Sanitary Engineering, California State Board of Public Health, June 26, 1943.

(30) **Disposal of Sewage into the Atlantic Ocean,** by A. D. Weston, *Modern Sewage Disposal*, pp. 209–218 (1938).

(31) **Congress Authorizes Ohio and Potomac River Compacts,** J. G. Ellis, *Water Works and Sewerage*, July, 1940, Vol. 87, p. 320.

(32) **Connecticut State Water Commission,** Ninth Biennial Report for Years 1940–1942 (1942).

(33) **The Stream Control Commission, State of Michigan,** Sixth Biennial Report for Years 1941–1942 (1943).

(34) **Wartime Construction Adds over 645 Sewage Works,** by M. M. Cohn, *Sewage Works Engineering*, February, 1943, Vol. 14, pp. 76–78.

(35) **Interim Report, Committee on Sewage Disposal,** Langdon Pearse et al., Am. Public Health Assn., Oct. 12, 1943.

(36) **Some New Research and Test Data on Grease Interceptors,** by F. M. Dawson and A. A. Kalinske, *Plumbing and Heating Business*, February, 1942. See also: *Plumbing and Heating Business* for November and December, 1939; also: Discussion by Dawson, *Proceedings*, Am. Soc. C. E., November, 1942, Vol. 68, pp. 1627–1628.

(37) **Are You Losing War Materials and Dollars Down Your Sewer?** by E. N. Mortenson, *National Provisioner*, April 24, May 1, 1943.

(38) **How New York Utilizes Sewage Scum,** by N. I. Kass, *American City*, November, 1943, Vol. 58, No. 11, pp. 56–57.

(39) **Wartime Operation and Maintenance Problems of Sewage Disposal Plants,** by A. P. Learned, *Public Works Engineers Yearbook*, 1943, pp. 197–201.

(40) **Rendering of Sewage Grease at Civilian and Army Plants,** by C. P. Gunson, *Sewage Works Engineering*, July, 1943, Vol. 14, pp. 329–330.

(41) **War Production Grease Sold to Renderer at Denver,** by C. P. Gunson, *ibid.*, May, 1942, Vol. 13, pp. 253–254.

(42) **Annual Report of the Bureau of Sewers, Worcester, Mass. Year Ending December 31, 1942,** by J. H. Brooks, Jr.

(43) **The Grease Problem in Sewage Treatment,** by A. L. Fales and S. A. Greeley, *Proceedings*, Am. Soc. C. E., February, 1942, Vol. 68, pp. 193–209; 269–279; May, 1942, Vol. 68, pp. 804–808; June, 1942, Vol. 68, p. 1071; September, 1942, Vol. 68, pp. 1178–1180; November, 1942, Vol. 68, p. 1627; March, 1943, Vol. 69, pp. 425–426.

(44) **Standard Methods for the Examination of Water and Sewage,** Am. Public Health Assn. and A.W.W.A., 8th Edition, 1936.

(45) **Sewage Treatment in the United States.** U. S. Public Health Service. *Public Health Bull. 132* (1923).

(46) **A Rapid Method for Determining Grease in Sewage and Sludges,** by H. W. Gehm, *Water Works and Sewerage*, August, 1943, Vol. 90, pp. 305–308.

(47) **Oil and Grease in Sewage,** by W. S. Mahlie, *Sewage Works J.*, May, 1940, Vol. 12, pp. 527–556.

(48) **Grease Extraction from Sewage and Sludge,** by R. Eliassen and H. B. Schulhoff, *ibid.*, May, 1943, Vol. 15, pp. 491–498.

(49) **Grease in Sewage, Sludge and Scum. I. Determination of Total and Non-saponifiable Grease,** by H. W. Gehm and E. H. Trubnick, *ibid.*, May, 1941, Vol. 13, pp. 467–484.

(50) **Determination of Grease in Sewage, Sludge and Scum. II. Determination of Grease in Sewage,** by H. W. Gehm, *ibid.*, September, 1941, Vol. 13, pp. 927–935.

(51) **Grease in Sewage, Sludge and Scum. III. The Separation of Grease from Sewage by Mechanical and Chemical Methods,** by H. W. Gehm, *Sewage Works J.*, July, 1942, Vol. 14, pp. 799–810.

(52) **Grease in Sewage, Sludge and Scum. IV. Transformation of Grease in Activated Sludge,** by H. Heukelekian, *ibid.*, May, 1943, Vol. 15, pp. 499–511.

(53) **Waste Disposal,** by F. W. Mohlman, *Chemical and Metallurgical Engineering*, January, February, 1943, Vol. 50, pp. 78–81, 127–131.

(54) **Industrial Wastes in Wartime,** by F. W. Mohlman, *Sewage Works J.*, November, 1943, Vol. 15, pp. 1164–1181.

(55) **Surveys of Liquid Wastes from Munitions Manufacturing,** by R. S. Smith and W. W. Walker.
I. TNT Wastes.
II. Smokeless Powder Wastes.
III. Small Arms Ammunition.
IV. Tetryl Wastes.
V. Nitroglycerine Wastes.
Public Health Reports, September 10 and 17, Vol. 58, pp. 1365–1379; 1393–1412.

(56) **Disposal of Distillery Slop,** by H. P. Bassett, *Chemical and Metallurgical Engineering*, December, 1938, Vol. 45, p. 673.

(57) **Complete Treatment of Distillery Wastes,** by A. M. Buswell and M. Le Bosquet, *Industrial and Engineering Chemistry*, July, 1936, Vol. 28, pp. 795–797.

(58) **Industry Abates Stream Pollution,** by H. H. Black and C. W. Klassen, *Sewage Works Engineering*, February, March, and April, 1941, Vol. 12, pp. 74–79; 170–174; 215–220; 235.

(59) **From Waste Liquors to War Alcohol,** by J. R. Callahan, *Chemical and Metallurgical Engineering*, December, 1943, Vol. 50, pp. 104–107.

(60) **Sanitation Facilities for Military Posts,** by S. A. Greeley and E. S. Chase, *Civil Engineering,* July, 1942, Vol. 12, pp. 359–362.

(61) **Sewage Treatment Plants for National Defense Projects,** by S. A. Greeley and E. S. Chase, *Public Works,* Vol. 72, February, 1941, pp. 9–10; 39–41.

(62) **Designing Sewage Treatment Plants for Army Camps,** by W. E. Stanley, *Sewage Works Engineering,* May, 1942, Vol. 13, pp. 240–245.

(63) **Sewage Disposal Problems at Army Camps,** by P. Hansen and K. V. Hill, *Sewage Works J.,* March, 1942, Vol. 14, pp. 294–301.

(64) **Engineering Manual,** War Department, Chief of Engineers (1942).

(65) **Demountable Army Camp Plants of Substitute Materials,** by W. Johannesen, *Sewage Works Engineering,* January, 1943, Vol. 14, pp. 13–14.

(66) **Flexible Sewage Treatment for Army Camps,** by R. Eliassen, *Civil Engineering,* December, 1941, Vol. II, pp. 707–710.

(67) **High-degree Treatment for Camp Sewage,** by H. F. Porter, *Engineering News-Record,* January 14, 1943, Vol. 130, pp. 38–42.

(68) **Army Bio-filter Sewage Plant Provides Unusual Flexibility,** *ibid.,* July 30, 1942, Vol. 129, pp. 173–175.

(69) **Emergency Treatment for Army Sewage,** by F. Bachman, *ibid.,* July 16, 1942, Vol. 129, p. 107.

(70) **Sewage Treatment at Army Posts,** by L. H. Kessler and J. T. Norgaard, *Sewage Works J.,* January, 1942, Vol. 14, pp. 757–783.

(71) **Operation of Army Sewage Treatment Works,** by L. H. Kessler and J. T. Norgaard, *ibid.,* November, 1942, Vol. 14, pp. 1228–1263.

(72) **Water Demands and Sewage Production in Military Cantonments,** by S. M. Ellsworth, *American J. Public Health,* January, 1942, Vol. 32, pp. 21–27.

(73) **Maintenance of Electric Equipment in Sewage Plants,** by H. V. Crawford, Michigan Eng. Experiment Station, *Bull. 98* (Vol. 19, No. 1) pp. 5–12 (1943).

(74) **Conservation of Centrifugal Pumps,** by B. L. Cody, *Chemical and Engineering News,* December 25, 1943, Vol. 21, pp. 2103–2106.

(75) **Maintenance of Sewage Pumps,** by H. J. Meeker, *Sewage Works Engineering,* June, 1943, Vol. 14, pp. 274–277.

(76) **Equipment Maintenance in Time of War,** by M. M. Cohn, *ibid.,* October, November, December, 1943, Vol. 14, pp. 469–478; 534–539; 555–556; 586–590.

(77) **Wartime Substitute Manhole Covers and Frames,** by G. E. Symons, *Water Works and Sewerage,* May, 1943, Vol. 90, pp. 179–181.

(78) **Cleaning Encrusted Pipe Lines and Vacuum Filters,** by K. L. Mick, *ibid.,* July, 1942, Vol. 89, p. 314.

(79) **Tests in the Cathodic Method of Protecting Steel Water Tanks,** by G. L. O'Brien, *ibid.,* pp. 285–291.

(80) **Corrosion of Sludge Digester Parts Corrected by Application of Cathodic Protection,** by G. A. Parkes, *Water Works and Sewerage,* June, 1943,

Vol. 90, Reference and Data Section, pp. 250–254; *Sewage Works J.*, January, 1941, Vol. 13, pp. 48–60.

(81) **Corrosion Troubles in the Digester,** by R. Reed, *The Clarifier*, May–July, 1942, Vol. 7, pp. 5–7 (1942); *Public Health Engineering Abstracts*, January, 1943, Vol. 23, p. S:9.

(82) **Progress Report of Committee on Corrosion,** W. F. Schade et al., Ohio Conference on Sewage Treatment, 15th, pp. 90–98 (1941).

(83) **Sanitary Sewage Flows from Industrial Plants,** by R. S. Smith and W. W. Walker, *Sewage Works J.*, May, 1943, Vol. 15, pp. 525–530.

(84) **Sewage Disposal Design for Wartime Industrial Plants,** by M. H. Klegerman, *Engineering News-Record*, October 8, 1942, Vol. 129, pp. 492–495.

(85) **Another Proportional-Flow Weir; Sutro Weir,** by E. A. Pratt, *Engineering News*, August 27, 1914, Vol. 72, pp. 462–463.

(86) **History of Sutro Weirs,** by C. H. Capen, *Engineering News-Record*, October 21, 1943, Vol. 131, p. 585; Cf. *ibid.*, November 18, 1943, Vol. 74, p. 735.

(87) **A Proportional-Flow Weir,** by E. W. Rettger, *Engineering News*, June 25, 1914, Vol. 71, pp. 1409–1410.

(88) **Grit Chamber and Pump Station, Albany Sewage Works,** Part I, by J. H. Gregory, *ibid.*, June 22, 1916, Vol. 75, pp. 1164–1166.

(89) **A Weir with Flow Proportional to Head,** by E. G. Smith, *Engineering News-Record*, July 29, 1943, Vol. 131, pp. 198–199.

(90) **Grit Chamber Design,** by T. R. Camp, *Sewage Works J.*, March, 1942, Vol. 14, pp. 368–381.

(91) **New Activated Sludge Treatment Plant, Newark, N. Y.,** by G. D. Holmes, *Water Works and Sewerage*, April, 1933, Vol. 80, pp. 159–162.

(92) **The Elimination of the Detritus Dump,** by C. B. Townend, Public Works, Roads and Transport Congress, Paper No. 26, 1937; Abstract in *Surveyor*, Vol. 102, p. 137 (1937).

(93) **Grit Removal Methods at Olean, N. Y. Treatment Plant,** by N. M. Fuller, *Sewage Works Engineering*, November, 1942, Vol. 13, p. 562; *Water Works and Sewerage*, June, 1943, Vol. 90, Reference and Data Section, p. 187.

(94) **A Contribution to the Design of Grit Chambers,** by H. Blunk and G. P. Edwards, *Sewage Works J.*, May, 1933, Vol. 5, pp. 509, 522.

(95) **Apparatus for and Method of Separating Suspended Solids from Liquid by Sedimentation or Settling,** U. S. Patent No. 1,968,031, Issued July 31, 1934, to W. J. Donohue.

(96) **Operation of the Multiple Tray Primary Sedimentation Units,** by J. K. Frei, J. Missouri Water and Sewerage Conference, April, 1941; *Sewage Works Engineering*, August, 1941, Vol. 12, pp. 423–425.

(97) See Ref. 5, *Sewage Works J.*, Vol. 14, pp. 323–4 (1942).

(98) **W. A. Gilchrist. Settler,** U. S. Patent No. 1,960,904, Issued May 29, 1934. **Apparatus for Clarification,** U. S. Patent No. 2,003,357, Issued June 4, 1935.

(99) Data from U. S. Public Health Service Census, Supplemented by Cohn (34) and others.

(100) **Modified Sewage Aeration,** Part I, by L. R. Setter, *Sewage Works J.*, July, 1943, Vol. 15, pp. 629–641.

(101) **Load Distribution in the Activated Sludge Process,** by J. E. McKee and G. M. Fair, *ibid.*, January, 1942, Vol. 14, pp. 121–146.

(102) **Tallmans Island Works Opens for World's Fair,** by R. H. Gould, *Municipal Sanitation*, April, 1939, Vol. 10, pp. 185–212.

(103) **Activated Sludge Process for Treating Sewage and Apparatus Therefor,** by R. H. Gould, U. S. Patent No. 2,337,384, Issued December 21, 1943.

(104) **Operating Experience in New York City,** by R. H. Gould, *Sewage Works J.*, January, 1942, Vol. 14, pp. 70–80.

(105) **Studies on the Activated Sludge Process at Two Rivers, Wis., Part II, Stage Addition of Activated Sludge,** by C. N. Sawyer and G. A. Rohlich, *Sewage Works Engineering*, May, 1943, Vol. 14, pp. 234–238.

(106) **Improved Final Settling Tanks at Bowery Bay,** by R. H. Gould, *Civil Engineering*, June, 1943, Vol. 13, pp. 279–281; *Water Works and Sewerage*, April 1, 1943, Vol. 90, pp. 133–136.

(107) **Servicing of Porous Air Diffusers,** by F. C. Roe, *ibid.*, June, 1943, Vol. 90, Reference and Data Section, pp. 266–268.

(108) **Ohio Conference on Sewage Treatment,** 17th, Abstract by G. E. Symons, *ibid.*, August, 1943, Vol. 90, pp. 309–312.

(109) **Maintaining Open Diffuser Plates with Chlorine,** by R. B. Jackson, *ibid.*, September, 1942, Vol. 89, pp. 380–382.

(110) **Cleaning Diffuser Plates in Treatment Plants,** *Sewage Works Engineering*, August, 1943, Vol. 14, pp. 379–380.

(111) **Economics of Cleaning Air-Diffusion Media,** by J. J. Wirts, *Water Works and Sewerage*, June, 1943, Vol. 90, pp. 205–208.

(112) **Diffuser Plate Cleaning Versus Compressed Air Cost,** by W. F. Schade and J. J. Wirts, *Sewage Works J.*, January, 1942, Vol. 14, pp. 81–96.

(113) **Two-Stage Aeration,** by C. A. Scott, *Water Works and Sewerage*, September, 1942, Vol. 89, p. 407.

(114) **Annual Report, Rivers Department of City of Manchester, England,** Year ending March 30, 1927, Appendix II, Two-stage Process, pp. 42–47.

(115) **Activated Sludge Experiments at the Calumet Sewage Treatment Works,** by F. W. Mohlman and C. E. Wheeler, *Sewage Works J.*, October, 1930, Vol. 2, pp. 529–548.

(116) **Process for Treating Sewage to Purify the Same,** H. O. Halvorson and R. Smith, U. S. Patent No. 2,141,979, Issued December 27, 1938.

(117) **Apparatus for Biologic Purification of Liquids,** H. N. Jenks, U. S. Patent No. 2,168,208, Issued August 1, 1939.

(118) **The Present State of Biofiltration,** by A. J. Fischer, *Water Works and Sewerage*, October, 1942, Vol. 89, pp. 416–425.

(119) **Aero-Filter Results—Predictions and Performances,** by J. A. Montgomery, *Sewage Works Engineering,* November, 1942, Vol. 13, pp. 553–562.

(120) **A Discussion of High Capacity Filters with Special Reference to Aero-Filters,** by J. A. Montgomery, *Sewage Works J.*, September, 1941, Vol. 13, pp. 905–917.

(121) **Sewage Treatment,** O. Ward, U. S. Patent No. 2,258,398, Issued October 7, 1941.

(122) **High Daily Rate Trickling Filter Performance,** Report made under the Direction of the Board of Health Commissioners, Upper Mississippi River Basin Sanitation Agreement, by G. Walton, L. F. Warrick, and J. Wilson (1943).

(123) **High Rate Biological Sewage Treatment,** by S. A. Greeley, *Sewage Works J.*, November, 1943, Vol. 15, pp. 1062–1087.

(124) **The Treatment of Diluted Sewage by Bio-aeration, in the Presence of Nitric and Nitrous Nitrogen,** by J. H. Kershaw and J. Finch, *Institute of Sewage Purification,* Part II, pp. 397–408 (1936).

(125) **The Cyclo-Nitrifying Filter,** by J. H. Edmondson and S. R. Goodrich, *ibid.*, November 24, 1943.

(126) **Minneapolis-St. Paul Sanitary District,** Annual Reports 1939 to 1942, inclusive.

(127) **Effect of Temperature on Sludge Concentration,** by W. Rudolfs and R. P. Logan, *Sewage Works J.*, September, 1943, Vol. 15, pp. 894–907.

(128) **Treatment of Supernatant Digester Liquor,** by W. Rudolfs and L. L. Faulk, *Public Works,* Vol. 74, August, 1943, pp. 13–15.

(129) **Improving Operation and Reducing Costs at the Princeton Sewage Treatment Plant,** by I. R. Riker, *ibid.*, Vol. 72, July, 1941, pp. 14–16.

(130) **Sludge Digestion Temperature Control with Live Steam,** by A. M. Rawn, *Water Works and Sewerage,* June, 1942, Vol. 89, pp. 310–313; Reference and Data Section, June, 1943, Vol. 90, pp. 246–248.

(131) **Theory and Practice of Sewage Sludge Digestion Tank Heating,** by N. C. Wittwer, *Water Works and Sewerage,* June, 1943, Vol. 90, pp. 229–239.

(132) **Sludge Digestion, Design Studies, Observations and Developments at Cleveland, Ohio,** by W. E. Gerbell, *ibid.*, January, 1942, Vol. 89, pp. 409–415.

(133) **Five Years of Sludge Digestion Studies in Cleveland,** by J. J. Wirts, Ohio Conference of Sewage Treatment, Twelfth Annual, pp. 40–52 (1938).

(134) **Supernatant Stored above Sludge in Two-storied Digesters,** by H. W. Taylor, *Sewage Works Engineering,* January, 1942, Vol. 13, pp. 8–9.

(135) **Principles and Factors Influencing Vacuum Filtration of Sludge,** by A. L. Genter, *Sewage Works J.*, November, 1941, Vol. 13, pp. 1164–1208.

(136) **Rotary Vacuum Filtration of Sludge and the Effect of War on Operation,** by J. T. Lynch and U. T. Mann, *Sewage Works J.*, September, 1943, Vol. 15, pp. 807–816.

(137) **Minneapolis-St. Paul Sanitary District, Tenth Annual Report** (1942).

(138) **Filtration of Sewage Sludge at Neenah-Menasha, Wisconsin,** by J. M. Holderby, *Sewage Works J.*, September, 1941, Vol. 13, pp. 866–873.

(139) Discussion by A. L. Genter, *Proceedings*, Am. Soc. C. E., 1943, Vol. 69, pp. 323–332.

(140) **Operating Results and Experiences, Sewage Treatment Plant, Birmingham, Michigan,** by S. J. Mogelnicki, *Sewage Works J.*, September, 1941, Vol. 13, pp. 945–954.

(141) **Third Year of Sewage Treatment Operation at Buffalo,** by C. R. Velzy, J. W. Johnson and G. E. Symons, *Public Works,* Vol. 73, February, 1942, pp. 26–30.

(142) **Abbreviated Aeration at Cleveland Southerly,** by G. E. Flowers, *Sewage Works Engineering*, October, 1941, Vol. 12, pp. 543–547.

(143) **Westerly Sewage Treatment Plant, Cleveland, Ohio,** by W. E. Gerbel, *Sewage Works J.*, January, 1943, Vol. 15, pp. 104–109.

(144) **Metropolitan District in Hartford County, Connecticut, Operation of Sewage Treatment Plant, Report for Years 1941–1942.**

(145) **Filtration of Sewage Sludge at Lansing, Michigan,** by G. Wyllie, *Sewage Works J.*, September, 1941, Vol. 13, pp. 874–878.

(146) **Marion, Ind. Operating Report, Year ended August 31, 1941,** by D. Backmeyer, *ibid.*, May, 1942, Vol. 14, pp. 722–725.

(147) **Sludge Filtration of Muskegon,** by C. T. Mudgett, *ibid.*, May, 1941, Vol. 13, pp. 879–884.

(148) **Balancing Sludge at Richmond-Sunset Plant, San Francisco,** by B. Benas, *Sewage Works Engineering*, October, 1941, Vol. 12, pp. 534–537.

(149) **Springfield, Mass. Annual Report** (1941).

(150) **Sludge Drying and Fertilizer Production at Chicago Calumet,** by L. M. Johnson, *Sewage Works Engineering*, February, 1942, Vol. 13, p. 71.

(151) **Dewatering, Incineration, and Use of Sewage Sludge: A Symposium,** *Proceedings*, Am. Soc. C. E., 1943, Vol. 69, January, pp. 41–104; February, pp. 323–332; June, pp. 907–910; September, pp. 1055–1062.

(152) **Experience with Sludge Elutriation at the Metropolitan District Sewage Treatment Plant, Hartford, Connecticut,** by G. H. Craemer, *Sewage Works J.*, May, 1942, Vol. 14, pp. 621–628.

(153) **The Vacuum Filtration of Elutriated Sludge,** by C. E. Keefer and H. Kratz, Jr., *ibid.*, September, 1934, Vol. 6, pp. 845–896.

(154) **What Elutriation of Sewage Sludge Can Do,** by C. G. Weber, *American City*, Vol. 58, October, 1943, pp. 91–93.

(155) **About Pre-aeration. Its Function in a Sewage Works,** by H. W. Taylor, *ibid.*, Vol. 57, November, 1942, No. 11, pp. 57, 67.

(156) Discussion on Ref. 102, by L. S. Kraus, pp. 145–146.

(157) **Grease Removal at Army Sewage Treatment Plants,** by R. Eliassen, Federation Sewage Works Associations, 1943; Abstract in *Water Works and Sewerage,* December, 1943, Vol. 90, pp. 453–454.

(158) **Grease in Sewage Sludge and Scum. Part III. The Separation of Grease from Sewage by Mechanical and Chemical Methods,** by H. W. Gehm, *Sewage Works J.,* July, 1942, Vol. 14, pp. 799–810.

(159) **Compressed Air Method of Removing Scum from Sedimentation Tanks,** by E. J. Hamlin, *Engineering and Contract Record,* Vol. 55, p. 14 (1942).

(160) **Improved Sewage Clarification by Pre-flocculation Without Chemicals,** by A. J. Fischer and A. Hillman, *Sewage Works J.,* March, 1940, Vol. 12, pp. 280–306.

(161) **Minneapolis-St. Paul Sanitary District. Seventh Annual Report,** p. 45 (1939).

(162) **Flocculation of Sewage With and Without Chemicals,** by E. F. Leist, Ohio Conference on Sewage Treatment, Fifteenth, pp. 69–74 (1941).

(163) **Vacuum Flotation of Sewage and Industrial Waste,** Michigan Eng. Experiment Station, by A. J. Fischer, *Bull. 98* (Vol. 19, No. 1), pp. 13–28 (1943).

(164) **Vacuum Flotation,** by A. J. Fischer, *Engineering News-Record,* February 25, 1943, Vol. 130, pp. 57–61.

(165) **Availability of Chemicals for Civilian Uses,** *Sewage Works Engineering,* October, 1943, Vol. 14, p. 506.

(166) **Third Year of Sewage Treatment Operation at Buffalo,** by C. R. Velzy, J. W. Johnson and G. E. Symons, *Public Works,* Vol. 73, January, 1942, No. 1, pp. 27–28, 50; February, 1942, No. 2, pp. 27–30.

(167) **Results of Effluent Chlorination at Cleveland,** by J. J. Wirts, Convention of Federation of Sewage Works Associations, 1942.

(168) **Toronto Contracts for Sewage Treatment Plant,** *Public Works,* Vol. 74, November, 1943, No. 11, p. 40.

(169) **The Operation and Control of Activated Sludge Sewage Treatment Works,** Committee on Sewage Disposal, Am. Public Health Assn., L. Pearse et al., *Sewage Works J.,* January, 1942, Vol. 14, pp. 3–69.

(170) **Control of Activated Sludge Bulking by Chlorination,** by E. E. Smith, *Water Works and Sewerage,* June, 1943, Vol. 90, pp. 209–210.

(171) **Chlorination as an Aid in Control of Bulking,** by J. R. Turner, *Sewage Works Engineering,* March, 1942, Vol. 13, p. 145.

(172) **Chlorination of Sewage and Industrial Waste,** by H. A. Faber, Convention of Federation of Sewage Works Associations (1943).

(173) **Industrial Wastes in Wartime,** by L. W. Van Kleeck, Convention of Federation of Sewage Works Associations (1943).

(174) **Control of Chlorination,** Committee Report, C. K. Calvert et al., *J. Am. Water Works Association,* Vol. 35, 1943, pp. 1315–1328.

(175) **Design of Sewage Treatment Plants to Facilitate Operation,** by S. E. Coburn, *Sewage Works J.,* March, 1942, Vol. 14, pp. 257–264.

(176) **A Method of Destroying or Preventing the Growth of Algae and Certain Pathogenic Bacteria in Water Supplies,** by G. T. Moore and K. F. Kellerman, U. S. Dept. Agriculture, Bureau of Plant Industry, *Bulletin 64* (1904); also *Bulletin 76* Supra (1905).

(177) **Copper Sulfate as a Germicide,** by G. A. Johnson and W. R. Copeland, *J. Infectious Diseases I*, Supplement No. 1, p. 327 (1905).

(178) **The Disinfection of Sewage Effluents for the Protection of Public Water Supplies,** by K. F. Kellerman, R. W. Pratt and A. E. Kimberly, U. S. Dept. Agriculture, Bureau of Plant Industry, Bulletin No. 115 (1907).

(179) **Report on the Sewage Disposal Problem of New Haven,** Made to the Mayor, by C. E. A. Winslow and F. W. Mohlman (1918).

(180) **Wartime Industrial Wastes—Good and Bad Copper Affects Digestion—Iron Improves Settling at Kenosha,** by H. T. Rudgal, *Sewage Works Engineering*, February, 1943, Vol. 14, pp. 94–96, 103.

(181) **Industrial Wastes Affect Sludge Digestion, Eighth Biennial Report,** Connecticut State Water Commission, *Sewage Works Engineering*, October, 1941, Vol. 12, pp. 548–549.

(182) **Inhibition of Nitrification by Chromates,** by S. H. Jenkins and C. H. Hewitt, *Nature*, Vol. 147, pp. 239–240 (1940).

(183) **The Disposal of Waste Acids,** by W. W. Hodge, *Wire and Wire Products*, January, 1940; Abstract in *Sewage Works J.*, May, 1942, Vol. 14, pp. 736–737.

(184) **Recovery of Free Acid from Pickling Liquors,** by H. W. Gehm, *Industrial and Engineering Chemistry*, Vol. 35, pp. 1003–1004 (1943).

(185) P. de Lattre, British Patent No. 491,640 (1938).

(186) **A Laboratory Study of the Guggenheim Bio-Chemical Process,** by E. B. Phelps and J. G. Bevan, *Sewage Works J.*, January, 1942, Vol. 14, pp. 104–120.

(187) **Sewage Treating Process,** by C. C. Hays, U. S. Patent No. 1,991,896, Issued February 19, 1935.

(188) **Contact Aeration for Sewage Treatment,** by L. R. Griffith, *Engineering News-Record*, January 28, 1943, Vol. 130, pp. 138–142.

(189) **Contact Aeration for Sewage Treatment,** by J. Hurley, *The Surveyor*, Vol. 102, pp. 183–185 (1943); Abstract, *Engineering News-Record*, June 17, 1943, Vol. 130, p. 895.

(190) E. B. Mallory, U. S. Patent No. 2,154,132, Issued April 11, 1939.

(191) **Oxidized Sludge Process and the Equilibrium Index,** by E. B. Mallory, *Water Works and Sewerage*, August, 1941, Vol. 88, pp. 333–344.

(192) **The Combined Complete Treatment of Medium and High Concentration Wastes,** by E. B. Mallory, *ibid.*, April, 1942, Vol. 89, pp. 143–155.

(193) **The Oxidation Index,** by E. B. Mallory, *Water Works and Sewerage*, January, 1943, Vol. 90, pp. 31–40; also Michigan Eng. Experiment Station, *Bull. 95* (Vol. 18, No. 1), pp. 45–68 (1942).

(194) **Industrial Waste Treatment Practice,** by E. F. Eldridge, pp. 74–78, McGraw-Hill Book Co., Inc. (1942).

(195) **Operating Results in Lansing's Combined Garbage-Sewage Disposal,** by W. R. Drury, *Public Works Engineers Yearbook,* 1942, pp. 188–200; *Public Works,* Vol. 73, January, 1942, No. 1, pp. 24–26.

(196) **Dual Disposal at Two Midwestern Cities,** by S. L. Tolman, *Sewage Works Engineering,* November, 1943, Vol. 14, pp. 529–533.

(197) **Garbage Disposal in Sewage at Midland, Michigan,** by P. Stegeman, Michigan Eng. Experiment Station, *Bull. 95* (Vol. 18, No. 1), pp. 10–16 (1942).

(198) **The Economics of Sludge Gas Engines,** Committee on Sewerage and Sewage Disposal, A. P. Learned et al., *Public Works Engineers Yearbook,* 1941, pp. 264–270; *Sewage Works Engineering,* July, 1941, Vol. 12, pp. 365–366.

(199) **Symposium. Do Gas Engines Pay?** *Sewage Works Engineering,* July, 1941, Vol. 12, pp. 375–396.

(200) **Scrubbing Sewage Gas,** by H. E. Norris, *Water Works and Sewerage,* February, 1943, Vol. 90, pp. 61–64.

(201) **Iron Sponge as an Aid to Sewage Gas Engine Operation,** by R. W. Stafford, *Sewage Works Engineering,* July, 1942, Vol. 13, pp. 343–344.

(202) **Aurora Studies Corrosiveness of Sulfur in Digester Gas,** by W. A. Sperry, *ibid.,* January, 1943, Vol. 14, p. 24.

(203) **Excess Sludge Gas for Motor Truck Fuel. Report on Clayton Plant, Atlanta, Ga.,** Prepared by Georgia State Eng. Experiment Station, *ibid.,* January, 1942, Vol. 13, pp. 15–17.

(204) **Sludge Gas for Motor Fuel.** *Engineering News-Record,* December 4, 1941, Vol. 127, pp. 802–803.

(205) **Sedimentation and Digestion in Germany,** by K. Imhoff, *Modern Sewage Disposal,* pp. 47–49 (1938).

(206) **Sewage Treatment in Germany,** by A. Heilmann, *ibid.,* p. 334 (1938).

(207) **Sludge as Fertilizer in the War Emergency,** by L. W. Van Kleeck, *Water Works and Sewerage,* May, 1943, Vol. 90, pp. 177–178.

(208) **Sewage Treatment Plant Operations under Wartime Conditions,** by W. W. Mathews, Federation of Sewage Works Associations, 1943, *ibid.,* November, 1943, Vol. 90, p. 393.

(209) **Have Victory Gardens Produced a Greater Market for Sludge?** *Sewage Works Engineering,* September, 1943, Vol. 14, pp. 425–426, 439.

(210) **Dried Sludge Sold as Fertilizer Base,** by W. G. Willard, *ibid.,* May, 1942, Vol. 13, pp. 246, 266–267.

(211) **Chemical Composition of Sewage Sludges, with Particular Reference to Their Phosphoric Acid Contents,** by W. Rudolfs and H. W. Gehm, *Bulletin No. 699,* N. J. Agricultural Experiment Station (1942).

(212) **Riboflavin in Sewage Sludge,** by L. S. Kraus, *Sewage Works J.,* July, 1942, Vol. 14, pp. 811–817.

(213) **Bibliography of References to the Literature on the Minor Elements and Their Relation to Plant and Animal Nutrition,** by L. G. Willis, Fourth Supplement to 3rd Edition, 1943, published by Chilean Nitrates Education Board, Inc.

(214) **Fertilizers and Food Production,** by F. E. Bear, *American Fertilizer*, Vol. 99, September 11, 1943, pp. 5–7.

(215) **The Utilization of Sewage Sludge as Fertilizer,** Report of Committee on Sewage Disposal, Am. Public Health Assn., 1937; *Sewage Works J.*, November, 1937, Vol. 9, pp. 861–912.

(216) **Sewage Sludge Sales and $ $ $ Returns,** by R. W. Frazier, *Water Works and Sewerage*, March, 1942, Vol. 89, pp. 118–120.

(217) **Sewage Treatment at Appleton, Wis.,** by C. O. Baitz, *Water Works and Sewerage*, June, 1942, Vol. 89, pp. 273–274.

(218) **Fertilizers and Fertilizer Chemicals in Wartime,** by C. J. Brand, *Chemical and Engineering News*, Vol. 21, pp. 1788–1792 (1943).

(219) **Preliminary Observations on the Effect of Sewage Treatment Processes on the Ova and Cysts of Intestinal Parasites,** by W. H. Wright, E. B. Cram and M. O. Nolan, *Sewage Works J.*, November, 1942, Vol. 14, pp. 1274–1280.

(220) **The Effect of Various Treatment Processes on the Survival of Helminth Ova and Protozoan Cysts in Sewage,** by E. B. Cram, *ibid.*, November, 1943, Vol. 15, pp. 1119–1138 (1943).

(221) **Purification of Sewage from Cysts of Intestinal Parasites,** by E. Gordon, *Med. Parasit. and Parasitic Dis. Moscow*, Vol. 10, No. 2, pp. 236–243 (1941); *Tropical Diseases, Bulletin*, April, 1943, Vol. 40, p. 311; *Public Health Engineering Abstracts*, October, 1943, Vol. 23, p. 558.

(222) **Recommendations of Joint Committee on Rural Sanitation—Rural Sewage Disposal,** *Public Health Reports*, March 12, 1943, Vol. 58, pp. 417–448.

(223) **Patent Law for Chemists, Engineers and Students,** by C. H. Biesterfeld, John Wiley & Sons, Inc. (1943).

(224) **Patents, What a Business Executive Should Know About Patents,** by R. S. Hoar, The Ronald Press Co., N. Y. (1926).

(225) **Textbook on the Law of Patents,** by A. H. Walker, Walker-Bellers Edition, 4 Vol., Baker, Voorhis and Co., N. Y.

(226) **Preventive Medicine and Hygiene,** by M. J. Rosenau, Sixth Edition, D. Appleton-Century Company, Inc., pp. 363–374; 434–435 (1935).

(227) **Poliomyelitis in the United States in 1942 and a Summary of Its Prevalence from 1933 to 1942, Inclusive,** by C. C. Dauer, *Public Health Reports*, June 18, 1943, Vol. 58, pp. 937–945.

(228) C. Kling, A. Petterson and W. Wernstedt, *Communications, Inst. Med. Etat Stockholm* 3: 5 (1912).

(229) **An Epidemiological Study of Poliomyelitis,** by W. A. Sawyer, *American Journal Tropical Diseases and Preventive Medicine*, Vol. 3, p. 164 (1915).

(230) P. H. Harmon. *Journal American Medical Association*, Vol. 109, p. 6 (1937).

(231) **Poliomyelitis Virus in Human Stools,** by J. D. Trask, J. R. Paul and A. J. Vignec, *Journal Experimental Medicine*, June, 1940, Vol. 71, pp. 751–763.

(232) **Untreated Human Stools as a Source of Poliomyelitis Virus,** by H. A. Howe and D. Bodian, *Journal Infectious Diseases*, Vol. 66, pp. 198–201 (1940).

(233) **Poliomyelitis Virus in Sewage,** by J. R. Paul, J. D. Trask and C. S. Culotta, *Science*, Vol. 90, p. 258 (1939).

(234) **Observations on Fecal Examinations in Poliomyelitis,** by J. D. Trask and J. R. Paul, *Am. J. Public Health*, March, 1941, Vol. 31, pp. 239–244.

(235) **The Virus of Poliomyelitis in Stools and Sewage,** by J. R. Paul and J. D. Trask, *Am. Medical Association*, Vol. 116, pp. 493–498 (1941).

(236) **Hypothetical Relationship of Water Supplies to Poliomyelitis,** by K. F. Maxcy, *Am. J. Public Health*, January, 1943, Vol. 33, pp. 41–45.

(237) **Effect of Chlorination of City Water on Virus of Poliomyelitis,** by J. E. Kempf and M. H. Soule, *Proceedings*, Soc. Experimental Biology and Medicine, Vol. 44, pp. 431–434 (1940).

(238) **The Effect of Hypochlorites on the Virus of Poliomyelitis,** by J. E. Kempf, M. E. Pierce and M. H. Soule, *J. Bacteriology*, Vol. 41, p. 51 (1941).

(239) **The Significance of the Finding of the Virus of Infantile Paralysis in Sewage,** A Review by K. F. Maxcy and H. A. Howe, *Sewage Works J.*, November, 1943, Vol. 15, pp. 1101–1114.

(240) **Effect of Activated Sludge Process on Poliomyelitis Virus,** by H. J. Carlson, G. M. Ridenour and C. F. McKhann, *Sewage Works J.*, July, 1943, Vol. 15, pp. 673–678; *Am. J. Public Health*, September, 1943, Vol. 33, pp. 1083–1087.

(241) **Poliomyelitis Remains an Enigma, Editorial,** *Am. J. Public Health*, November, 1943, Vol. 33, pp. 1358–1359.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

BY L. STANDISH HALL

L. STANDISH HALL,[13] M. AM. SOC. C. E.[13a]—By assembling the available data bearing on all phases of the subject of reservoir sedimentation, Mr. Witzig has rendered a valuable service to the profession.

Under the heading, "Origin and Nature of Silt," Mr. Witzig describes sheet and bank erosion as the principal sources of reservoir silting. The relative importance of these two factors on any watershed depends largely on the gradient of the main channel, although in some instances other effects may govern. In streams having a flat slope, the sediment carried by the tributaries will be dropped in the main channel resulting in aggradation. Then, reservoir sedimentation is caused by a reworking of these sediments in the stream channel. When the slope of the main channel is steep, the sediment fed into it by the tributaries will be carried along by the velocity of the current and the dynamic action of the water will cause excessive bank and bed erosion by abrasive action. In such cases, the erosion effect of the sediment is cumulative, increasing as the water proceeds downstream until a saturation of the sediment load is reached.

Other sources of erosion not mentioned by the author, but quite prominent in the Coast Range of California, are landslides and earth flow, particularly when these occur along stream channels. The toe of a slide which encroaches on the waterway is eroded by the flowing water causing a progressive movement of the slide into the channel. When landslides are not cut by water courses, the erosion of the slide may be negligible. In such cases, the landslip merely results in the mass movement of the soil from a higher to a lower level (59).[13b]

Erosion from highway construction is also an important factor. Much of the current highway practice of discharging water from small culverts with a

NOTE.—This paper by Berard J. Witzig was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Joe W. Johnson; October, 1943, by John W. Stanley, Stafford C. Happ, and Thomas H. Means; November, 1943, by Carl B. Brown, and C. S. Jarvis; December, 1943, by Hugh Stevens Bell, and Harry F. Blaney; and March, 1944, by A. L. Sonderegger.

[13] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[13a] Received by the Secretary February 17, 1944.

[13b] Numerals in parentheses, thus: (59), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

free drop to the downstream end, or of failing to provide a basin at the outlet end of large culverts in which to dissipate the energy of the high velocity discharge through the culvert, has added many tons of sediment to stream channels.

Under the side heading, "Definition of Silt," the author cites a weight of 85 lb per cu ft for deposits in reservoirs. This value appears to be rather high unless the sediment in the reservoirs has been reworked and subjected to exposure and drying. Tests, by the writer, of samples of silty sand deposited by stream channels on their flood plains indicated a percentage of voids averaging 53% and a weight of the dry material of 77 lb per cu ft. For silty clay, the percentage of voids was 60% and the weight of the dry material 66 lb per cu ft. The weight of the dry material comprising the silt deposits in reservoirs, when not subjected to drying, is undoubtedly much less than that of material found in deposits on the overflow areas of stream channels. The void content of saturated sediments from reservoirs has been found by the writer to be as high as 80%, which corresponds to a dry weight of 33 lb per cu ft. The dry weight of deltaic deposits near the head of a reservoir, of course, would be greater and could equal the foregoing weight of the river bottom deposits, or in some instances it might be even greater.

Under the heading, "Transportation of Sediment to the Reservoir," the difference between the method of transportation of suspended sediment and of bed load is indicated by a comparison of Eqs. 1 and 5*a*. In the Kennedy formula, V_c varies directly as a power of the depth, whereas, in the Schoklitsch formula, since $V_c = \frac{0.00532\, D_g}{y_d\, S^{4/3}}$, it varies inversely with the depth. Hence, it may be inferred that, to keep fine sediment in suspension, the velocity must increase with the depth of the water. On the other hand, the movement of bed load, by which erosion the stream bed continually deepens the channel and makes the bank slopes steeper, is the result of drag. This drag, in accordance with the du Boys theory, is proportional to the depth of water and to the slope or gradient of the channel (60). The movement of fine sediment depends on the supply available, which is variable, and this movement in a large measure is independent of the volume of flow. Approximate curves may be developed, but the actual proportion of sediment is normally greater on a rising than on a falling stage and, in addition, varies greatly between individual storms for the same drainage area (61). On the other hand, the movement of bed load is directly related to the volume of flow and for a given stream channel can be predicted with greater accuracy.

Under the heading, "The Action of Sediment in Reservoirs," the author quotes Mr. Faris as follows: "* * * No suspended silt is carried through the reservoir and over the spillway until all of the clear water has been discharged." Observations of the writer indicate that this is not necessarily true in all instances. At the Pardee Dam on the Mokelumne River in California, the inflowing water is not of high turbidity at any time and during much of the year it is very clear. The maximum turbidity rarely exceeds 250 ppm. On one occasion in June, 1935, the sluice valves at the base of the dam had been opened and were discharging water of a turbidity of 20 to 30 ppm, while the

river flow was between 3,000 and 5,000 cu ft per sec. These sluices were closed for a period of about a week. In a few days, turbid water was observed to rise through the reservoir near the spillway and to pass over the weir lip. By opening the sluice valves again to reduce the spillway discharge to a very small

TABLE 7.—GENERAL CHARACTERISTICS, RESERVOIRS NEAR OAKLAND, CALIF.

Reservoir	Runoff, 1886–1887 to 1935–1936 (acre-ft per sq mile)	Drainage area (sq miles)	ORIGINAL STORAGE	
			Acre-ft	Acre-ft per sq mile, S_R
Chabot	409	42.05	15,860	377
Upper San Leandro	477	30.3	41,398	1,370
San Pablo	387	32.2	43,149	1,340
Temescal	409	2.75	656	238

TABLE 8.—SUMMARY OF SEDIMENTATION OF RESERVOIRS NEAR OAKLAND, CALIF.

Period	Interval, in years	Mean annual runoff (acre-ft per sq mile)	ANNUAL RATE OF SILTING			
			Mean annual (acre-ft)	Cu ft per acre	Total runoff (%)	Acre-ft per 100 sq miles
(*a*) CHABOT RESERVOIR						
1875–1900	25	447	60.4	98	0.32	144
1900–1910	10	526	131.0	212	0.59	312
1910–1923	13	393	37.5	61	0.23	89
1923–1942	19	232	17.3	28	0.18	41
1875–1942	67	387	54.2	88	0.33	129
(*b*) UPPER SAN LEANDRO RESERVOIR						
1924–1935	11	270	15.4	34	0.19	51
1935–1941	6	765	134.3	302	0.58	443
1924–1941	17	445	57.4	128	0.43	189
(*c*) SAN PABLO RESERVOIR						
1917–1936	19	226	12.4	26	0.27	38
1936–1938	2	732	378.5	820	1.60	1,210
1938–1943	5	616	111.0	234	0.56	345
1917–1943	26	340	64.8	137	0.59	202
(*d*) TEMESCAL RESERVOIR						
1869–1907	38	438	4.8	118	0.40	175

quantity, the release of the turbid water was transferred back to the sluice valves. The density currents in the reservoir at the time were such that the discharge of the turbid water could be transferred from the sluice valves at the base of the dam to the spillway, a vertical height of 327 ft, depending on the point at which the maximum volume of water was discharging.

In connection with the section of the paper on "Estimating the Rate of Reservoir Sedimentation," the writer can add results on sedimentation obtained from surveys of storage reservoirs of the East Bay Municipal Utility District near Oakland, Calif. Surveys of sediment deposits in one of these reservoirs have continued over a period of 67 years (62). Additional surveys of the silt deposits have been made during the last few years and the results of these and previous surveys are summarized in Tables 7 and 8. The runoff per square mile from these drainage areas is relatively small and, on the other

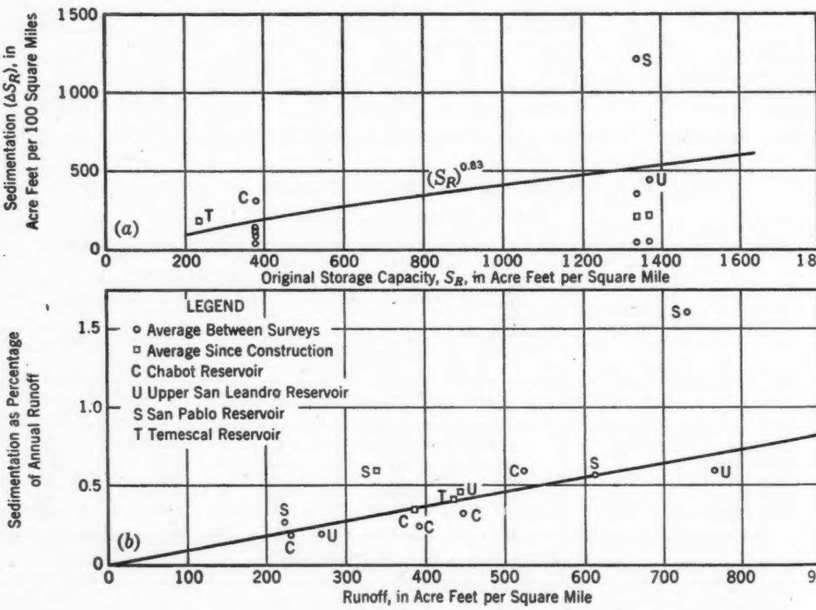


FIG. 8.—SEDIMENTATION OF RESERVOIRS NEAR OAKLAND

hand, the storage per square mile of drainage area is relatively large. Consequently, little runoff has been wasted over the spillways and the water that has been wasted was practically clear. Therefore, the deposits in the reservoirs represent all of the sediment produced on the watersheds during the periods recorded.

The relation between sediment deposited and the storage capacity per square mile of drainage area is plotted in Fig. 8 (in a manner similar to Fig. 3). The range in sediment deposits for the various periods between surveys is indicated. Based on the average deposit of sediment in the smaller reservoirs, Eq. 6 has been plotted using for x his value of 0.83. The application of this equation to the reservoirs listed in Tables 7 and 8 would require a much greater sediment deposit in the larger reservoirs than was actually recorded. The relation between the runoff by square miles of drainage area and the sediment deposits expressed as percentage of the volume of runoff has been plotted in Fig. 8(*b*). For these reservoirs, a straight-line relation appears to exist between

these two factors. In the San Pablo Reservoir, the high deposits recorded between 1936 and 1938 were due to road construction and land subdivision above the reservoir during a period of heavy runoff. The subsequent survey in 1943 indicates that the rate of erosion from this watershed had returned to that indicated in Fig. 8(*b*).

This relation between sediment deposits and volume of flow, if of general application, would be very useful in expanding short records of silting to indicate the true normal. Table 8 indicates that the rate of sediment deposits varies considerably during various runoff cycles. If the watershed erosion is proportional to the annual runoff, the mean rate of erosion could be determined from the ratio of the runoff during a period of sediment measurement to the long-time mean runoff from the watershed. Where appreciable quantities of sediment are discharged through sluiceways or over the spillways, this sediment should be combined with the quantity deposited in the reservoir to determine the entire watershed erosion.

Because of the relatively large size of the reservoirs of the East Bay Municipal Utility District near Oakland as compared with the discharge from the watershed, it has been estimated that their life will range from 500 to 800 years.

At Pardee Reservoir a record has been kept for several years of suspended sediment discharge through the sluiceways and turbines by taking samples of the river water immediately below the dam. The sediment discharge has been computed from these samples with a weight of 62.5 lb per cu ft for suspended material and a weight of 35 lb per cu ft for dry material if deposited in the reservoir. This light weight for deposited material has been used due to the fine colloidal nature of the sediment discharged through the sluiceways. The results of these measurements of the four years, 1940–1943, are as follows:

Year	Weight of discharge sediment, in tons	Equivalent volume, discharge in acre-feet
1940	4,806	6.3
1941	1,949	2.6
1942	6,951	9.2
1943	8,983	11.8

These quantities, added to the sediment deposits in the Pardee Reservoir, will give the total sediment load from the watershed. The drainage area above Pardee Reservoir is largely in granitic rock and the sediment load in streams is small. The mean annual runoff is 800,000 acre-ft from a drainage area of 575 sq miles, and the original storage capacity in the reservoir was 210,000 acre-ft. In the first ten years that the reservoir was in operation, the deposits were found to be approximately 1,000 acre-ft. Less than 10% of the total sediment was discharged through the sluice valves. The deposits consisted of about an equal proportion of delta deposits at the head reservoir, and of bottom-set beds of fine material in the main body of the reservoir. At this rate of sedimentation, the life of Pardee Reservoir will be longer than 2,000 years.

Eq. 7 expressing the relation between sediment deposits and the size of the reservoir appears to be defective in that it does not include the factor of runoff.

A more suitable equation might be:

$$\frac{\Delta S_R}{E} = f\left(\frac{S_R}{Q_a}\right) \qquad (20)$$

in which, in addition to the notation of the paper, Q_a is the mean annual runoff. Other factors enter the problem which are not included in Eq. 20, but usually these are of secondary importance. These factors include: (1) The size gradation of the sediment; (2) the flow characteristics of the stream entering the reservoir; (3) the shape of the reservoir; and (4) the manner of operation of the reservoir, particularly the discharge of water through sluice gates or over the spillway.

Under "Remedies for Reservoir Silting," Mr. Witzig has presented four methods. Of these, sluicing of water through sluice gates and erosion control on the watershed above the reservoir appear to be the only practicable and economical means. The latter procedure offers the greatest possibilities as it controls the sediment at its source. The several types of "upstream engineering" listed by the author have been practiced by the East Bay Municipal Utility District on its local watersheds near Oakland. This work has been conducted since 1939 in cooperation with the U. S. Soil Conservation Service (62*a*)(63)(64).

Important causes of reservoir sedimentation on many watersheds, in addition to those mentioned by the author, are landslides or earth flow, particularly where the earth is carried into the stream channel. This condition is cured by draining the slide. However, in some instances where the toe of the slide has come to rest in a stream channel, the construction of a debris barrier below the slide will check the downstream movement of the sediment from the slide and result in its eventual stabilization.

To be more completely effective, debris barriers in stream channels must have a relatively narrow discharge weir with respect to the width of the channel at the dam. Otherwise, only the bed load will be trapped, since the velocity of the current will be only slightly reduced and the time of ponding will be so short that most of the suspended sediment will pass over the spillway. The discharge of a considerable portion of the suspended load is a fault naturally inherent in these small ponds. The growth of vegetation along the stream channel will increase the efficiency of the barrier by facilitating the deposit of fine material. If properly designed and maintained, a debris barrier will trap sediment greatly in excess of its normal water-holding capacity at the time of construction. This is made possible by the deposition of material well above the elevation of the spillway lip. An important function of debris barriers on main channels is the reduction of bank scouring by flattening the stream grade, and the reduction of erosion by localizing the drops at dams where the energy of the water can be dissipated in properly designed stilling pools.

An aid in the planning of the control of erosion on watershed lands is the taking of water samples from various tributaries entering the reservoir during storms. Channels carrying excessive sediment can thus be segregated and the source of erosion located later during more favorable weather. The erosion control work can then be planned to accomplish the maximum results with a minimum expenditure.

Bibliography.—

(59) **"Landslides and Related Phenomena,"** by C. F. Stewart Sharpe, Columbia Univ. Press, New York, N. Y., 1938, Chapters 4 and 5.

(60) **"Le Rhone et les Rivières à Lit Affouillable,"** par M. P. du Boys, *Annales des Ponts et Chaussées*, Vol. XVIII, 1879, Pt. 2, p. 141.

(61) **"Field Practice and Equipment Used in Sampling Suspended Sediment,"** *Report No. 1* in **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** published by St. Paul U. S. Engr. Dist. Sub-office, Hydr. Laboratory, Univ. of Iowa, Iowa City, Iowa, August, 1940, pp. 85 and 90.

(62) **"Silting of Reservoirs,"** by L. Standish Hall, *Journal*, A.W.W.A., Vol. 32, No. 1, January, 1940, pp. 25–42. (*a*) pp. 39–42.

(63) **"Erosion Control for Reservoir Protection,"** by Harry E. Reddick, *Soil Conservation*, Vol. VII, No. 3, September, 1941, pp. 77–79.

(64) **"Drop Structures for Erosion Control,"** by L. Standish Hall, *Civil Engineering*, May, 1942, pp. 247–250.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

GEOLOGY IN HIGHWAY ENGINEERING

Discussion

By F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, and Jacob Feld

F. H. Kellogg,[5] M. Am. Soc. C. E.[5a]—In furnishing a comprehensive view of the diverse factors involving applications of geology to highway engineering, the author has made a valuable contribution. By treating the various headings in the paper from a standpoint of what is actually being done by highway engineering organizations, he approaches the subject from a practical standpoint. Considering the general nature of the paper, one can scarcely take issue with the statements and conclusions. However, a more definite indication of the limitations of geological work in this field would increase cooperation and reduce antagonisms engendered by divergent viewpoints. With no intention of trying to act as a qualified critic of geological methods, the writer suggests some possible limitations.

Obviously, as emphasized by the paper, the more time taken for a geological study, and the more drilling, laboratory, and geophysical information available, the more complete and reliable will be the conclusions. An illustration of the danger of geological study without a sufficient background of detailed exploration is furnished by the experiences in connection with the Moffat Tunnel in Colorado (33)(34).[5b] There, a condition not anticipated by preliminary investigation was a fault zone of "rotten granite," which tended to cave into the tunnel bore, costing a number of lives and raising the cost of the project from an estimated 7 million dollars to more than 17 million dollars. The complaint was made that, although the geologists had given an elaborate report, "the voice of prophecy was not upon them." Under more modern conditions, nobody would expect any prophecies, but a vastly greater amount of

Note.—This paper by Marshall T. Huntting was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by W. W. Crosby; and March, 1944, by Carl B. Brown.

[5] Senior Materials Engr., TVA, Knoxville, Tenn.

[5a] Received by the Secretary February 10, 1944.

[5b] Numerals in parentheses, thus: (33), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

time and money would be spent on gathering specific data on the rocks in the tunnel site before starting construction on a project of such magnitude.

There are specialists among geologists just as there are among civil engineers and among highway engineers, and so it is questioned whether many of the functions treated by Mr. Huntting could be performed by the average geologist. Particularly in connection with subgrades, slides, and ground waters, a wealth of firsthand experience is a prime requirement for most professional work, and an acquaintance with the elements of elasticity, hydrodynamics, and soil mechanics is often desirable. Doubtless, members of state geological surveys working closely with the highway departments, and some geological engineers, possess these qualifications. Many other geologists do not.

A type of rock slide which has been overlooked by geologists on some construction jobs occurred on the Madden Dam Project in Panama. A shallow key trench was cut in a bluff rising steeply from the Chagres River to a height of nearly 200 ft. The rock showed almost no signs of seams, crevices, or bedding planes. Soon after the excavation, however, it began to crack and masses of rock threatened to fall on the operations at the base of the bluff. The cracked material was removed until, once more, only smooth, solid rock was exposed. In another day or so, more cracks appeared and the danger of rock slides increased. The situation grew steadily worse until a geologist was called in consultation. He immediately diagnosed this condition as a case of overstressed rock. The very fact that the rock was so solid, so free of any strains which would have relieved the stresses, was the cause of the trouble. The geologist showed that the same rock across the river was well jointed, and never acted this way. Thereafter, no more rock was removed from that area without immediately replacing it with concrete, so that the stress balance was undisturbed, and no more "popping rock" was encountered. This case illustrates the value of a geologist with wide experience in practical excavation and construction problems.

An engineer dealing with rocks and soils must gain some knowledge of geological principles from the very nature of his work. A geologist could advance, substantially, the cause of understanding between the two professions by avoiding conclusions that would be apparent to any one with the slightest knowledge of geology. In general, delineation of alluvial and residual soils, lengthy observations as to the dangers of cuts through rocks with a shaley cleavage dipping toward the road, and detailed descriptions of readily identified rock formations and aquifers—all would fall in this category.

Mr. Huntting has mentioned the possibilities of the geological approach to problems of subgrades and slides in artificial fills. Although the writer does not believe that engineering work in soils must be a highly restricted specialty, or that any particular group has a monopoly on statistical development and mathematical physics, he contends that the value of the geological approach to these problems is limited. For example, in the paper under the heading, "Frost-Heave Problems," the statement is made that soils with a high clay content are likely to show considerable frost heaving because they have small pore spaces, which cause high capillary rise. On the other hand, clays usually have such low permeabilities that only a small quantity of water is involved

in this capillary action. Soils with pore spaces sufficiently small to permit a high capillary rise, yet large enough to involve appreciable quantities of water, are the worst offenders as regards frost heaving. These offenders can be identified quickly and economically by simple physical tests. Again, the statement is made in the paper under the heading, "Prevention and Correction of Landslides," that sands with rounded grains are less stable than those with angular grains. This is obvious, but it gives no basis for determining the slope required for a given fill under given conditions of compaction and pore pressure.

The writer once made a study of the effect of mineral composition on the mechanical properties of soils by concentrating individual minerals, grinding them to certain grain-size distributions in a colloid mill and performing various physical tests on the monomineralic soils thus produced. The minerals were selected with the help of C. S. Ross, of the U. S. Geological Survey, who pioneered in the field of clay minerals and their identification. Observing the tremendous effects of small admixtures of certain minerals in the finer fractions of a soil, and those of the intimate mixtures of various minerals in many of the soils studied, the conclusion was reached that a purely physical approach would be more rapid and practical for most control work. The complex petrographic and X-ray techniques involved in identification of the clay minerals do not favor the mineralogic approach to problems in soils engineering except in the fields of research and development. In these fields, this approach has great promise, particularly in connection with studies of chemical and physico-chemical processes (35)(36)(37). In applying the results of this research, however, rapid and simple physical tests based on performance experience appear preferable.

The foregoing comments seem warranted because there are engineers and geologists who would let geological analyses supplant, rather than supplement, purely physical investigations of the problems in the paper. Evidently Mr. Huntting is not of this opinion, and the writer strongly concurs with his basic conclusion that geology is a powerful tool in many highway engineering activities and that there is not a sufficiently general realization of this fact.

BERLEN C. MONEYMAKER,[6] ASSOC. M. AM. SOC. C. E.[6a]—The twelve problems listed and discussed in this paper include nearly all the major geologic problems involved in the construction and maintenance of highways. Many of the same geologic problems are also concerned with the construction and maintenance of railroads.

Mr. Huntting's treatment is rather general, on the whole, and few professional engineering geologists are likely to challenge, seriously, either his statements or his conclusions. Engineers, however, might be inclined to feel that he tends to "oversell" the importance of geology and to underestimate the engineer's knowledge of geologic problems. As most colleges and universities require their engineering students to complete satisfactorily one or more courses in geology, most engineers embark on their professional careers with some geologic background. Engineers who specialize in field work—constructing

[6] Chf. Geologist, TVA, Knoxville, Tenn.

[6a] Received by the Secretary March 6, 1944.

and maintaining highways, building dams, driving tunnels, etc., learn much about geologic problems by actual experience. Admittedly, some of this knowledge is gained the "hard way" and occasionally a disastrous failure results from ignorance of geologic conditions. Nevertheless, minor geologic problems, such as the determination of slopes in excavations, etc., may ordinarily be handled satisfactorily by engineers.

Mr. Huntting's paper raises a number of questions, among which are the following:

1. Should All the Work Included in the Twelve Problems Be Performed by Geologists?—These problems involve several distinct earth sciences, some of which are not geologic in the strictest sense, and it is debatable whether geologists should enter the fields of related or allied sciences. The Geologic Division of the Tennessee Valley Authority (TVA), for many years the largest staff of engineering geologists ever assembled, may be cited as an organization in which a high degree of specialization, even in geology, is recognized. The work done by this group is geologic in the very strictest sense, and work in the related or allied sciences is performed by other staffs of specialists. The actual testing of materials is left to materials testing specialists; the testing of soils is left to specialists in soil mechanics; geophysical work is left to geophysicists, and so on. The Geologic Division of the TVA is sufficiently large to include specialists in several branches of geology. Specifically, the activities of this staff include the following: Engineering geology for dam sites and tunnels; the location and geologic study of quarry sites; the collection of samples for testing; petrographic studies of foundation rocks, concrete aggregate, and riprap materials; study of ground-water problems in areas adjacent to reservoirs; examination and appraisal of mineral lands; the presentation of testimony in court at condemnation trials; and miscellaneous geologic work, such as studies of bridge foundations, deep highway and railroad cuts, and drainage projects for malaria control.

In the opinion of the writer, geophysics, soil mechanics, and the testing of materials should not be considered in the field of geology. However, many excellent geologists have become specialists in these related sciences.

2. How Should a State Highway Department Provide for Its Geologic Work?—As the author has shown definitely that geologic information may be used advantageously in the construction and maintenance of highways, it might seem that each state highway department should equip itself with a geologist or with a small geologic staff. However, the matter is not as simple as that. If geologic advice is to be of any great value, it must be that of a specialist. In this day of specialization, no geologist is sufficiently versatile to qualify as a specialist in each branch of the science involved in the author's twelve problems. It would be difficult to assemble a small staff of two or three men capable of even moderately reliable work on all these highway engineering problems. A well-rounded staff, qualified to perform specialized work in all branches of the earth sciences involved, would require the personal services of the following specialists, some of whom are not geologists: Engineering geologist, petrologist, ground-water hydrologist, economic geologist, materials

testing specialist, soil mechanics specialist, and geophysicist. Obviously, most state highway departments cannot justify the maintenance of such a large staff.

Although Mr. Huntting notes the close cooperation that exists between certain state highway departments and state geological surveys and mentions the engagement of special consultants, he makes no specific recommendation as to how a state highway department may best obtain geologic advice of high quality. Considering the great variety of geologic and related investigations required for the construction and maintenance of highways, the state geological surveys can afford the best and most economical assistance. In states where the geological surveys are small or inactive, the faculties of state universities may supply able geologic advisers.

3. Is the Geologic Examination Sufficient of Itself?—In some instances, a geologic examination may afford all the information necessary, but in many cases a mere examination is not sufficient.

A geologic examination of a tunnel site without a considerable amount of exploratory core drilling may not yield much precise information. The writer examined the Garzas-Peñuelas power tunnel site in Puerto Rico before construction was undertaken and was unable to predict the geologic conditions with desirable precision. However, a very thorough study of the stratigraphy and the orogenic and geomorphic history of the area enabled him to predict accurately the nature of the geologic problems.

The Garzas-Peñuelas tunnel is 11,674 ft long and extends in a north-south direction across the rugged Cordillera Central, which forms the insular divide of Puerto Rico. Rock decay has progressed to such great depths (200 ft) below the surface that bedrock was exposed only at the intake portal and in the channels of two small streams that cross the tunnel location. Moreover, the entire tunnel site was covered by a luxuriant growth of tropical vegetation. By a painstaking study of the residuum along the location, the character of the underlying rock could be predicted with accuracy. A study of the only rock exposures in the area revealed the location of a few faults. Although it was expected that hundreds of faults were to be crossed, there was no way of determining precisely either their location or magnitude or their importance as geologic problems.

Fortunately, little serious trouble was experienced in the actual driving of the tunnel but this was due to the excellence of the engineering supervision rather than to lack of difficulty. As predicted, hundreds of faults were encountered but most of them were small and presented no problems. Many of them were marked by shear zones and occasioned appreciable overbreakage. For support, these structures required timbering during construction and lining afterward. Two major faults, both apparently active, were crossed with some difficulty and after considerable delay. Both yielded copious quantities of water under considerable head. One of them resulted in the caving of the heading and the partial filling of the bore with 1,000 cu yd of fine gouge and 500 cu yd of broken rock. This fault zone was 152 ft wide and yielded 1,300 gal of water per min. The other fault also resulted in the failure of the heading and the partial filling of about 40 ft of the bore with broken rock and gouge. The fault zone was only 21 ft wide and yielded 200 gal per min of water under

a head of 160 lb per sq in. Five months were required to tunnel through the 173 ft of ground represented by these two fault zones.

At the time the geologic examination was made, the tunnel site was definitely fixed by engineering and other considerations, and preliminary construction activities were already under way. No geologic exploration of any type was made along the tunnel location. Although the geologic conditions could have been determined in advance only from drill holes on 25-ft centers, from a few feet to 640 ft deep along the tunnel site, the cost of the program would have been prohibitive.

The Garzas-Peñuelas tunnel is an instance in which the engineer in charge handled difficult practical geologic problems remarkably well.

The Moffat Tunnel through the continental divide in Colorado represents another instance in which a geologic examination, without exploration, was highly inadequate. Numerous other instances in which geologic examinations, without the advantage of exploration, proved inadequate or entirely misleading could be mentioned.

One geologic condition not mentioned by the author in his discussion is the silicosis hazard involved in the quarrying or tunneling of siliceous rocks. Rocks containing more than 10% free silica are likely to cause silicosis among workmen unless the dust count per cubic foot is kept below 10,000,000 particles of two microns (0.002 mm) or smaller in size. The free silica content of the rock to be tunneled or quarried may be readily determined by petrologic studies. The silicosis hazard is very largely eliminated by adequate ventilation, the use of wet drilling methods, and the spraying of the broken rock during such operations as loading and crushing. Failure to control siliceous dust invites serious trouble. The contractor on the Hawk's Nest Tunnel in West Virginia was sued by some 600 former employees who claimed to have contracted silicosis while at work on that project.

This paper is an excellent and timely contribution on the relation of geology to highway engineering. Geology is a valuable tool that is used advantageously in several branches of civil engineering. Its limitations, however, should be emphasized as strongly as its advantages. The most reliable geologic work, in any branch of the science, is done by a specialist, and even a specialist, in many instances, must rely on data made available by exploration.

E. F. Bean.[7,7a]—A brief paper cannot be comprehensive in coverage. Thus, the following notes reflect a desire for a longer paper rather than adverse criticism of this paper, which is an excellent review of the work that has been accomplished.

The "Wisconsin Materials Survey Procedure" would be more accurately described if the first sentence under that heading were omitted, and the following statement were added at the close of the paragraph:

> "This information makes it possible to let contracts at lower figures, or for the county to perform the work at lower cost since the uncertain factor, source of road aggregate, has been eliminated."

The State Highway Department does not own and operate any local plants.

[7] State Geologist, Geological and Natural History Survey, The Univ. of Wisconsin, Madison, Wis.

[7a] Received by the Secretary March 10, 1944.

The list of investigators of inferior aggregate in concrete (see heading, "Suitability of Various Earth Materials for Surfacing, Concrete Construction, and other Highway Uses") should be expanded to include, in addition to Mr. Stanton, A. T. Goldbeck, Stanton Walker, and W. J. Emmons, Members, Am. Soc. C. E., and Fred C. Lang and M. O. Withey.

Of course, the Bibliography cannot include all pertinent references but at least five others might well be included (38)(39)(40)(41)(42). Perhaps, also, reference should be made to the *Annotated Bibliography of Economic Geology*, published semiannually, which contains a section devoted to engineering geology.

A. T. Bleck,[8] Esq.[8a]—The conclusion of the paper is a very succinct summary. As the author states, highway engineering is sufficiently broad to include many varied kinds of work requiring specialized knowledge, which the engineer should avail himself of as the occasion arises.

In exploration for materials deposits, complicated bridge foundation problems, classification of excavation materials, tunnels, etc., consultation on geological formation appears to be an obvious need. However, considering problems arising in connection with the soil mantle of the earth, highway engineers in general possibly have relied on, or sought for, soil mechanics data to too great an extent and to the exclusion of other factors, such as the geological formation and subsequent development of soils. This opinion is based on not only personal observations but also the apparent controversial tenor of the literature on the general subject of soils in their relation to highway and airport construction.

If geology, to a reasonable degree of certainty, can be applied to predict the probable exposure to moisture to which the soil in given locations will be subjected, the engineer can then correlate these data with those supplied by the science of soils mechanics and effect an adequate design without sacrificing economy. Perhaps a greater mutual understanding, on the one hand, of the problems encountered, and on the other, of the knowledge and counsel that could be made available, is necessary and Mr. Huntting's paper seems at least to be a step in that direction.

Lyman W. Wood,[9] Esq.[9a]—The twelve problems, listed by the author, constitute an adequate statement of a geologist's duties with the Iowa State Highway Commission. In subsurface explorations of bridge sites, the procedure is perhaps more advanced than that used by other highway organizations. Test borings are made for all bridges where foundations are expected to extend below stream bed. The boring records are reviewed by a geologist and become part of the final bridge plan, supplemented in many cases by geological sections of the strata penetrated and by results of physical tests on samples of critical materials. Valuable information on excavation conditions, as well as on probable pile bearing, is thus made available to contractors.

[8] Constr. Engr., State Highway Commission of Wisconsin, Madison, Wis.

[8a] Received by the Secretary March 10, 1944.

[9] Formerly Geologist, Iowa State Highway Commission, Ames, Iowa.

[9a] Received by the Secretary March 10, 1944.

Road material resource surveys, to be of greatest value, must be continuous. The Iowa Highway Commission has followed a plan for periodic reexamination of all material deposits, with provision for revising estimates of quantity or commenting on quality, as advisable. The more important deposits are examined annually; others are examined less frequently. Proper maintenance of a file of material resource information in an up-to-date condition will require a considerable annual outlay of time, but experience indicates that the results are worth while.

PHILIP KEENE,[10] Assoc. M. Am. Soc. C. E.[10a]—In the United States, highway work has attained the category of "big business," and highway engineering is replacing many of its rule-of-thumb practices with scientific solutions. As the author has demonstrated, geology is an important scientific aid to proper design, construction, and maintenance of highways.

Probably geology in Connecticut highway work today is most useful in determining depths of the various soil strata overlying bedrock when making borings for bridges and structures and in soil surveys. As the author indicates, knowledge of the geology of a bridge site, before or after some borings are made, permits economical and judicious selection of subsequent bore hole locations and depths. Soil surveys have been made by the department since 1942 on all cuts and swamps, on the larger projects. The surveys in cuts are intended primarily to locate rock surface for determining probable yardages of rock and earth excavation, inclination of cut slopes, and width of right of way to be purchased; but they also enable the engineer to determine definitely, or tentatively, the need for subbase, underdrains, and slope drains, the suitability of soil for fills, and the treatment of special features which occasionally are encountered.

The bedrock and ledge of Connecticut consist chiefly of ancient complex metamorphic and igneous formations which are very difficult to decipher. However, covering a strip that is 15 to 25 miles wide and extends throughout the middle of the state in a north-south direction are traprock and sedimentary "brownstone" formations which form part of the Newark series of the Triassic Period. These formations dip from 15° to 30° in an easterly direction, forming definite characteristic ridges in numerous localities. In this Triassic strip the depths to rock surface can be predicted quite accurately for an area, especially on these ridges, before or after a bore hole has been completed. This characteristic dip also permitted definite calculation of the location of the contact plane between the traprock sill and the underlying red sandstone between bore holes for the proposed tunnel at West Rock, New Haven, Conn.

A use for geology, not mentioned by the author and needed only rarely, is in the examination and interpretation of soil or rock cores to determine the plane or zone of rupture of a slide. Such cores might have many small faults and distortions caused by old disturbances, in addition to those caused by the slide being studied. In such a case, geology would be indispensable in distinguishing between the old and the new movements.

---

[10] Soil Mech. Engr., State Highway Dept., Portland, Conn.

[10a] Received by the Secretary March 13, 1944.

JACOB FELD,[11] M. AM. SOC. C. E.[11a]—Undoubtedly a knowledge of geology is helpful in solving the twelve problems enumerated in this paper, most of which are specialized problems of soil mechanics. If the author's statements are taken at face value, soil mechanicians (or is it soil "mechanics") as well as foundation engineers are daily encroaching on a restricted field. Since soil control and analysis has always borrowed information from the science of geology, uses and has used knowledge contributed by the sciences of physics and chemistry, and also employs analogies from electricity and hydraulics, the encroachment on the field of geology is not unique.

To be expert enough to handle the twelve problems properly, the geologist should be a well trained highway engineer, a specialist in soil mechanics, an expert in tunnel design and construction, and a divining rod operator, as well as an expert witness. Might it not be simpler to take a highway engineer who has had the necessary training in soil mechanics and insist that he obtain sufficient training in geology than to expect the geologist to understand the engineering problems enumerated?

Suitable gravel pits generally may be located by the application of geology, but the writer has found in practice that the only way to know the characteristics of sand or gravel deposits is to take samples while digging is in progress. Even boring samples do not give the complete answer. On a recent large project where almost one million cubic yards of run-of-bank gravel was needed, approximate locations of available supply within a radius of ten miles had been determined from topographic and geological considerations. When the time came to use this gravel, almost every one of the locations was found to be a disappointingly thin cover of gravel overlying sand. In a number of locations the gravel was covered with glacial till, the cost of the removal of which made the supply of gravel very expensive.

The author's statements concerning the suitability of various earth materials (in which he includes concrete aggregates) fail to take into consideration the fact that standards have been set up by the American Society for Testing Materials, American Concrete Institute, the Bureau of Public Roads, and forty-eight state highway departments, in the form of specifications listing not only the requirements of physical and chemical characteristics but also, in great detail, the methods of testing the compliance with such requirements. Most of these specifications are based on an accumulation of empirical information which may be determined in some instances by geological considerations. However, the wealth of experience indicates that empirical data based on prior performance is a safer guide to the highway engineer.

On U. S. route No. 9, north of Peekskill, N. Y., there is a monument to a contractor who undertook the excavation through a hillside, definitely glacial in origin according to all geological information available, which, when exposed, was found to be a thin skin of glacial till and boulders overlying nice hard solid rock. The unclassified excavation unit price in the contractor's contract applied to the excavation of this cut, more than 100 ft deep in solid rock, put him in the class of an ex-contractor. In this instance, somewhat too great a re-

[11] Cons. Engr., New York, N. Y.

[11a] Received by the Secretary March 14, 1944.

liance on "known" geological characteristics of that area were disastrous, to the contractor.

Under the heading, "Prediction of Character of Material to Be Excavated," Mr. Huntting states:

> "The cost of digging solid rock may be ten times or more the cost of digging an equal amount of earth (Fig. 1); * * *."

In more than one instance, the writer has had actual experience, where the cost of digging rock would have been much less than that of the type of earth encountered. The writer also wishes to dissent from the statement (see heading, "Competency of Materials for Bridge Foundations") that:

> "The superstructure of a bridge can be no more sound than its piers and abutments, whose soundness in turn is dependent upon their design and upon the nature of the rock or soil foundations on which they rest (Fig. 2)."

The soundness of a superstructure has no relation to the type of rock or soil encountered in the foundations.

Under the heading, "Prevention and Correction of Landslides," some mention should be made of the slide that occurred in Pittsburgh, Pa., in 1922 and also of the landslides in Los Angeles, Calif., and Fort Peck, Mont.

When discussing bridge foundations, the author seems to indicate that taking and testing soil samples is entirely the work of geologists. In all his soil mechanics studies on that subject, the writer has never heard this claim.

The writer's experience in 1943, when searching for an available supply of gravel, afforded an interesting side light on the evaluation of mineral lands with gravel pockets. He found buying farms, including the houses and livestock, less costly than buying gravel pits. No amount of test pit probing or boring data would give a true indication of the amount of gravel available, except at a most unreasonable cost. Evaluating the land based on expected yield of gravel involved so many factors of both time and exploratory costs that it was much more expedient to buy the land. For sites where agreement as to the amount of available gravel could not be reached, experts were called in to analyze the situation from the geologist's point of view. The result was testimony in the form of reports of one geologist against another; and, as in the litigation examples given in the paper, one geologist gave testimony which was accepted and the other geologist gave testimony which was not accepted. So the score was even.

The writer's objection to this paper is again demonstrated by the "Conclusion." The attempt to delegate all questions "dealing with the earth materials and the forces that act upon them" to geologists, on the plea that highway engineering is sufficiently broad to include this overspecialization work by geologists, seems an attempt at overspecialization.

A quotation from a paper written by Edwin G. Conklin (43) is quite pertinent:

> "The extreme specialization of science has tended to obscure the larger aspects of Nature, as a whole. It has been said that modern science consists in knowing more and more of less and less. Certainly it is concerned largely with analysis and always more analysis.

"Of course, analysis is necessary, but so also is synthesis if we are to see organisms as living beings. We lose sight of these larger aspects of life in the process of analysis, unless we reverse this procedure from time to time and consider organisms and environment synthetically.

"Life has been compared to a beautiful tapestry, woven in intricate design of many threads and colors. By means of physics, chemistry, physiology, anatomy, embryology and genetics we unravel this texture, separate its constituent threads and colors, but lose the pattern as a whole. These analytical sciences have enormously increased our knowledge of life's constituent elements and processes, but the pattern of the tapestry is usually neglected or ignored."

In a similar sense, the writer makes a plea that the subject of highway engineering be considered as a unit and, if the highway engineer needs training in the science of geology to properly solve the problems encountered, he should have that training. If he delegates the soil problems to geologists, why not also delegate such chemical problems as those of cement and its uses to chemists and static problems on the design of pavements and bridges to physicists, and thus leave no place for the highway engineer.

Bibliography.—

(33) **"Construction Methods on the Moffat Tunnel,"** by R. H. Keays, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 63–112.

(34) **"Completion of Moffat Tunnel of Colorado,"** by Clifford Allen Betts, *ibid.*, Vol. 95 (1931), pp. 334–371.

(35) **"Physico-Chemical Testing of Soils and Application of the Results in Practice,"** by Hans F. Winterkorn, *Proceedings*, Highway Research Board, Vol. 20, 1940, pp. 798–806.

(36) **"Application of Base Exchange and Soil Physics to Problems of Highway Construction,"** by Hans F. Winterkorn, *Proceedings*, Soil Science Soc. of America, Vol. I, 1937, pp. 93–99.

(37) **"Surface Chemical Factors Influencing the Engineering Properties of Soils,"** by Hans F. Winterkorn, *Proceedings*, Highway Research Board, Vol. 16, 1936, p. 293.

(38) **"The Mechanics of Frost Heaving,"** by Stephen Taber, *Journal of Geology*, Vol. 38, 1930, pp. 303–317.

(39) **"Physics of the Earth,"** National Research Council, Div. of Physical Sciences, Pt. 9—"Hydrology," by O. E. Meinzer, 1942.

(40) **"Engineering Geology,"** by H. Ries and T. L. Watson, John Wiley & Sons, Inc., New York, N. Y., 1936.

(41) **"Geology in Engineering,"** by C. P. Berkey, *Mineralogist*, Vol. 9, 1941, pp. 327–328 and 352–353.

(42) **"Geology and Its Application to Pennsylvania Turnpike,"** by Arthur B. Cleaves, *Monthly Bulletin*, Pennsylvania Dept. of Internal Affairs, November, 1940.

(43) **"Ends as Well as Means in Life and Evolution,"** by Edwin G. Conklin, *Transactions*, New York Academy of Sciences, February, 1944, p. 125.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SURGES IN PANAMA CANAL REPRODUCED IN MODEL

Discussion

By Harold A. Weggel

Harold A. Weggel,[6] Jun. Am. Soc. C. E.[6a]—It is very interesting to note the close conformity of model observations with prototype Tests Nos. I, II, III, and V as shown in Figs. 3 and 4. However, close conformity in five observations does not necessarily mean that all conclusions based on the model would hold true in the prototype. Test No. III and Test No. V are of special interest because they illustrate the results of consecutive lock operations; but deductions are incomplete unless the data are available for the periods of synchronization and anti-synchronization.

Observations made by the U. S. Geological Survey on the Ohio River near Huntington, W. Va., in 1938 between lock 27 and lock 28 indicate the difficulties of predicting the heights of translatory waves. At times of low discharge on the Ohio River, the slightest change of discharge, caused by operation of lock chambers and controls on the dams both upstream and downstream, was enough to create a surge. Occasionally, anti-synchronization of lock operations resulted in negative slope readings, and at other times similar operations would be the cause of superelevated translatory waves. Not always is it the amount of change of discharge that determines the height of wave or depth of trough. The rate of change of discharge may be the prime factor, especially on closing operations in conjunction with reflected waves.

The time of travel of translatory waves for the 4.65 miles between the gages at lock 28 and at 24th Street is between 17 min and 21 min; and the time of travel for the 10.6 miles between lock 27 and lock 28 (and return) is between 88 min and 100 min. Theoretical calculations give the following results for $V = \sqrt{g\,y}$ (Eq. 1): For the upper reach with $y = 10.0$ ft, $V = 17.95$ ft per sec; and, for the lower reach with $y = 13.9$ ft, $V = 21.2$ ft per sec. The time of travel is 19.3 min for the lower reach and 29.1 min for the upper reach or

Note.—This paper by F. W. Edwards and Edward Soucek was published in January, 1944, *Proceedings*.

[6] Associate Structural Engr., Diamond Magnesium Co., Painesville, Ohio.

[6a] Received by the Secretary February 28, 1944.

48.4 min for the entire distance. At times of low flow no difference was noticed between the time of wave travel downstream and upstream.

If the canal surges in the Panama Canal are comparable to those on the Ohio River, it is doubtful whether any model could be accurate enough to reproduce the surges that would be encountered in the operation of the proposed larger lock. The only possible method is the study of the prototype itself with sufficient gages installed to plot the passage of translatory waves and to correlate the effects with the causes. When the causes of the disturbing waves are determined, it should be easy to eliminate the waves through control of the causes.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW

Discussion

BY A. A. KALINSKE, AND EDWARD H. TAYLOR

A. A. KALINSKE,[8] ASSOC. M. AM. SOC. C. E.[8a]—Hydraulic engineers are reminded, by this paper, of an item which, although of minor significance in most practical varied-flow problems, is nevertheless of considerable interest, because it reemphasizes the difference between the two fundamental methods of attacking hydraulic problems—namely, the energy and momentum principles. Nothing new is really presented since, as the author mentions at the end of his paper, this particular problem has been very completely discussed in papers by Messrs. Bakhmeteff[6] and Keulegan.[7] Nevertheless, it is well that the author has brought it more definitely to the attention of hydraulic engineers.

The writer would like to reinterpret and restate certain ideas mentioned or implied in the paper. First, the fundamental reason that the wrong application of Bernoulli's equation is made to stream cross sections having nonuniform velocity distribution is a faulty understanding of the derivation of Bernoulli's equation. Basically, the expression known as Bernoulli's equation is the integral along a streamline of the Euler differential equation for steady frictionless fluid motion.[9] Euler's equation is obtained from a simple application of Newton's law ($f = m\,a$) to a fluid particle. Of course, since basically the energy and momentum laws stem from Newton's law, these can be used to derive Bernoulli's equation. Because, fundamentally, Bernoulli's equation holds only along a streamline, its use cannot be extended directly to a finite stream cross section except for the special case in which the velocity of all the fluid particles in the cross section of a stream tube is the same.

NOTE.—This paper by William S. Eisenlohr, Jr., was published in January, 1944, *Proceedings*.

[8] Associate Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[8a] Received by the Secretary February 7, 1944.

[6] "Coriolis and the Energy Principle in Hydraulics," by B. A. Bakhmeteff, *Theodore von Karman Anniversary Volume*, Contributions to Applied Mechanics and Related Subjects, 1941, p. 59.

[7] "Equation of Motion for the Steady Mean Flow of Water in Open Channels," by Garbis H. Keulegan, Research Paper RP1488, *Journal of Research*, National Bureau of Standards, Vol. 29, July, 1942, pp. 97–111.

[9] "Fundamentals of Hydro- and Aero-Mechanics," by L. Prandtl and O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 112.

If an equation of the Bernoulli type is to be used for stream cross sections having nonuniform velocity distribution, it is necessary to derive such an expression from basic dynamic considerations. In any such derivation, of course, the analyst must consider all frictional forces since these are always present for real fluids. This was done by Mr. Keulegan,[7] using both the energy and momentum principles. The author has derived similar expressions in a somewhat less clear, and less rigorous, manner.

The energy principle, as used by Mr. Keulegan and the author, probably could be called the "power" principle since the terms calculated are really the "flow" of energy per unit time and the work done per unit time. The important item to recognize in the use of this principle is that all external and internal energy losses and work done must be taken into account. In using the momentum law only external forces need be considered. Hydraulic engineers will recognize this basic difference in the use of these two principles in the analysis of the hydraulic jump.

In a truly rigorous sense, Eq. 8*a* or Eq. 16*c* should not really be called a Bernoulli equation. Such an equation is basically true only along a streamline, and was originally derived for ideal fluids. Eq. 8*a* is actually a momentum equation and Eq. 16*b* a power equation.

The author's analyses, in reality, are concerned with deriving varied-flow equations for open channels. However, neither Eq. 8*b* nor Eq. 16*c* could be used for solving varied-flow problems (for example, obtaining backwater curves), except for very short increments of the distance, L. The reason for this is that the equations have not been integrated with respect to L; instead average values have been used for such variables as A and wall shear. Distance L should have been designated as dL.

The author's statements regarding Eq. 2 and Eq. 3 are basically inaccurate. He states that $a = \frac{u^2{}_B - u^2{}_A}{2\,L}$, which, of course, comes from the relation $a = \frac{u\,du}{dL}$ when integrated between points A and B, assuming a as constant. Obviously, there is no reason to assume a as constant in any length of channel. It would have been better if Eq. 5 had been written as:

$$(dp)\,(dA) + \gamma\,(dL)\,(dA)\sin\theta - df_{r,m} = \rho\,(dA)\,u\,(du) \quad \text{.......(18)}$$

Any attempt to integrate Eq. 18 with respect to L (which the author really does) would indicate that an assumption should be made regarding the variation of A with L. The author assumes, of course, that such variation is negligible; this is inconsistent since he does not at the same time assume variation in u as negligible. Similar weaknesses are present in the derivation of Eq. 16*c*, involving the energy principle.

Reference to Mr. Keulegan's handling of this same problem indicates that his final equations are in differential form as far as the distance L is concerned. However, the author's results (which show that the coefficient C_m should be used when only external forces are being calculated, and that C_e should be used when the total energy loss is concerned) agree with Mr. Keulegan's final conclusions. This is the all-important part of the paper and is correct, even

though the derivations leading up to these facts are quite inelegant mathematically, and not overly convincing.

It is hoped that this paper will clarify the question of which "velocity-head" correction coefficient should be used, and why, when applying the Bernoulli equation to various practical fluid flow problems. Many engineers have not been too clear in their minds regarding this matter and have accepted textbook statements without a careful analysis of the dynamics involved. A more thorough consideration of classical hydrodynamics and the exercise of greater care in transferring relations developed therein into the field of real fluids would have prevented the various misunderstandings such as have occurred in the practical use of the Bernoulli equation.

EDWARD H. TAYLOR,[10] JUN. AM. SOC. C. E.[10a]—Certain assertions in this very interesting paper are somewhat novel, and their acceptance would require a change in the thinking methods of the writers of "standard textbooks on hydraulics." The following statement leading to Eqs. 2 and 3 was of particular interest to the writer:

> "The basic equation by the momentum theorem or energy principle is that of Newton's second law of motion, $F = m\,a$, wherein the substitution $a = \dfrac{u^2_B - u^2_A}{2\,L}$ has been made. The difference between the two methods lies in which side of the equation the distance L is placed."

Since momentum and energy are basically different physically—that is, momentum is a vector quantity and energy a scalar quantity—it appears incorrect

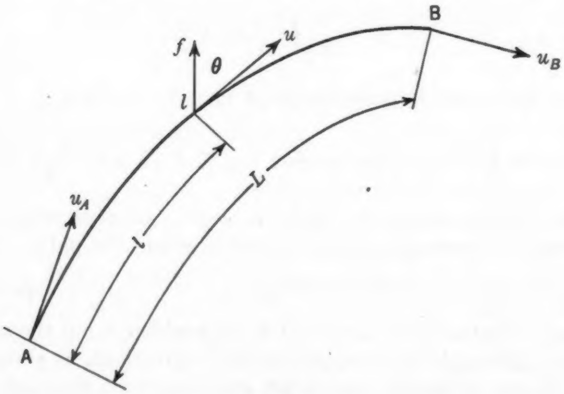


FIG. 3

to state that one can be obtained from the other by a mere transposition of one factor.

In other words, the appearance of the length L on one side of the equation or the other should make no difference in the fundamental nature of the equation itself. The F in Eq. 2 and the F in Eqs. 3 are apparently identical.

[10] Asst. Prof. of Civ. Eng., Univ. of Arizona, Tucson, Ariz.

[10a] Received by the Secretary March 1, 1944.

It might be of value to recall that, in a given mechanical system, the force that causes a change in momentum is not, in general, the same force that causes a change in kinetic energy. Forces normal to the path of motion have no effect on the kinetic energy, but definitely influence the momentum. As a simple example, consider a mass particle fastened to a point by a string and describing plane uniform motion about the point.

The key to this situation lies in the substitution, $a = \frac{u^2{}_B - u^2{}_A}{2\,L}$. Consider a particle (fluid or otherwise) which follows the path shown in Fig. 3. Here f is the resultant of all forces acting on the particle when it occupies the position l. The equation of motion is:

$$f \cos \theta = m \frac{du}{dt} \qquad (19)$$

Multiplying both sides of Eq. 19 by dl,

$$f \cos \theta \, dl = m \frac{du}{dt} \, dl \qquad (20a)$$

or

$$f \cos \theta \, dl = m \, u \, du \qquad (20b)$$

Integrating

$$\int_0^L f \cos \theta \, dl = m \left(\frac{u^2{}_B}{2} - \frac{u^2{}_A}{2} \right) \qquad (21)$$

An average force F may be defined by

$$F = \frac{1}{L} \int_0^L f \cos \theta \, dl \qquad (22)$$

provided that its nature be carefully kept in mind. Then,

$$F\,L = m \left(\frac{u^2{}_B - u^2{}_A}{2} \right) \text{; or (see Eq. 2) } F = m \left(\frac{u^2{}_B - u^2{}_A}{2\,L} \right).$$

Thus, the F of Eq. 2 does not seem to be the resultant force acting on the mass, but rather an average component taken along the path. As far as the writer is aware, the use of the substitution $a = \frac{u^2{}_B - u^2{}_A}{2\,L}$ permits no other interpretation. Furthermore, the right-hand member is not the rate of change of momentum (although it is its dimensional equivalent, as shown), but it is the average change in kinetic energy per unit length on the path L. Rate of change of momentum is a vector quantity, by Newton's second law, and Eq. 2 contains no element of direction.

Thus, the writer is led to the conclusion that Eq. 2 is truly an energy equation. This would eventually lead to the further statement that energy coefficients should be used in Eq. 7. The foregoing remarks, although contradictory to the author's general thesis, are in no way intended to be derogatory. They are merely an expression of the writer's understanding of the meaning of momentum and energy.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

Discussion

By Portland P. Fox, and J. K. Black

Portland P. Fox,[5] Assoc. M. Am. Soc. C. E.[5a]—Because it offers remedies for serious defects in dam foundations, this well prepared paper by Mr. Hays should be of considerable interest to other engineers and geologists. The writer succeeded Roger F. Rhoades as geologist on the Kentucky project in April, 1941, and logged approximately the lower 50 ft of the deep channel. Although most of the exploration was done while Mr. Rhoades was geologist at the project, the writer had the opportunity to study the excavation at close hand.

The selection and exploration for the Kentucky dam site involved many unusual geologic problems and conditions.[6] Kentucky Dam is located at the northeastern edge of the Mississippi embayment and there are no exposures of bedrock on the left bank of the river for many miles either upstream or downstream. Bedrock outcrops near the right bank in the vicinity of the dam and consists of a few small, scattered, flat-lying limestone beds. It was impossible, therefore, to make any accurate detailed predictions of the bedrock conditions to be expected below the river bed from what could be seen on the surface. The territory surrounding Kentucky Dam is of very low relief; the highest hills, located from 1 to 6 miles away from the river, reach a maximum height of only 200 ft. Both abutments and most of the surrounding region are capped by from 10 to 100 ft of Cretaceous sand, clay, and gravel, which largely concealed most of the bedrock. Before and after the deposition of the sand, clay, and gravel, the limestone bedrock had been subjected to a long period of intense and

Note.—This Symposium was published in November, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by Berlen C. Moneymaker.

[5] Geologist, TVA, Fontana Dam, N. C.

[5a] Received by the Secretary February 17, 1944.

[6] "Subriver Solution Cavities in The Tennessee Valley," by B. C. Moneymaker, *Journal of Geology*, Vol. XLIX, No. 1, January-February, 1941.

deep weathering.[7] During the late Pleistocene period[8] the river cut its bedrock channel 70 ft below the present river bed and later filled the channel with more recent gravel.

Between 1935 and 1937 seven dam sites on the lower Tennessee River, two on the Cumberland River, and four on the Ohio River were intensely investigated by numerous core drill holes. Most of the sites were found to be entirely unsatisfactory because of the absence of rock to depths of 200 to 300 ft below river bed or because of cavernous conditions below the top of rock.

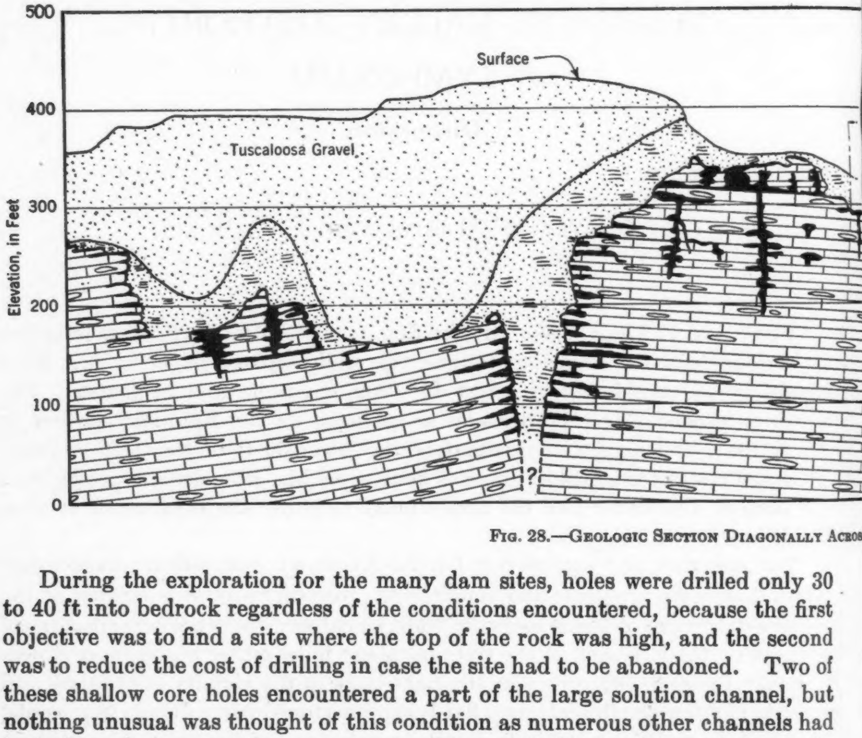


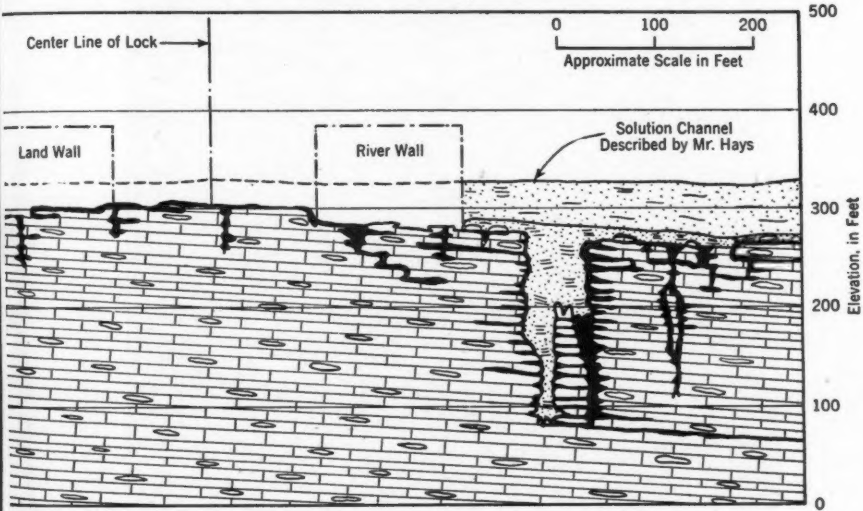
FIG. 28.—GEOLOGIC SECTION DIAGONALLY ACROSS

During the exploration for the many dam sites, holes were drilled only 30 to 40 ft into bedrock regardless of the conditions encountered, because the first objective was to find a site where the top of the rock was high, and the second was to reduce the cost of drilling in case the site had to be abandoned. Two of these shallow core holes encountered a part of the large solution channel, but nothing unusual was thought of this condition as numerous other channels had looked much worse on the core record. The vertical holes did not indicate the width of the channels. This fact was realized early and many of the exploration holes were drilled on a 45° angle to discover the width of the channels. Of the 8,500-ft axis, 90% was covered with holes at 45°; but the large channel occurred near the right abutment not far from the only rock outcrops at the site, and in an area not penetrated by angle holes. The angled core drill holes revealed a slightly smaller but similar solution channel between Station 38+00 and Station 40+00, or about in the middle of the first bottom, and under the west earth embankment. Because there was a thick cover of nearly 100 ft of overburden, this area was treated by closely spaced grout holes.

[7] "Relation of the Tuscaloosa Formation of Western Kentucky to a Pre-Existing Weathered Terrain," by R. F. Rhoades, *Bulletin*, Geological Soc. of America, Vol. LI, No. 12, Pt. 2, December 1, 1940, p. 1940.

[8] "Geologic Profile of the Lower Tennessee River," by Portland P. Fox, *Journal*, Tennessee Academy of Science, April, 1944.

Core holes drilled during the early exploration also disclosed a large escarpment in the bedrock which greatly influenced the location and design of the structure immediately upstream from the lock walls. This escarpment in the bedrock is approximately 100 ft high and has a north-south trend approximately parallel to the solution channel described by Mr. Hays. Recent exploration for grouting in the right abutment indicates that the escarpment is but one side of a larger solution channel that has formed in the crest of a breached anticline which is the dominant structure of the region (see Fig. 28).



Lock and Right Abutment, Facing Upstream

This channel has been explored down to El. 130, or 301 ft below the surface, where it is approximately 40 ft wide.

The bedrock underlying the lower 60 miles of the lower Tennessee River is known as the Fort Payne formation of Lower Mississippian age. Although it is essentially flat lying, broad gentle folds having maximum dips of 10° are common but not obvious. Deep drill holes at the site have proved the formation to be 545 ft thick and to have a remarkably uniform lithology throughout this entire thickness. To make the geologic problem more complicated, the lower part of the overlying Warsaw and St. Louis formations in the vicinity have lithologic characteristics similar to the Fort Payne formation. The Fort Payne formation consists of alternating thick beds of dense, dark gray, relatively pure limestone and of thinner layers, lenses, and nodules of dense, glassy black chert (see Fig. 29).

The chert comprises about one third of the formation and is entirely insoluble, but, due to its brittle nature and numerous joints, it readily breaks up into small angular blocks when weathered. Some of the layers of chert are more than 1 ft thick and such layers frequently project nearly 1 ft from the receding soluble limestone beds. The limestone beds contain some impurities in the form of finely divided silt, but normally they are more than 70% soluble.

Because of the roughness of the cavity walls formed by the overhanging layers of chert, the residual clays never completely fill the spaces immediately under the chert. Thus, the limestone walls are exposed to constant solution and as the chert layers break off the face slowly recedes.

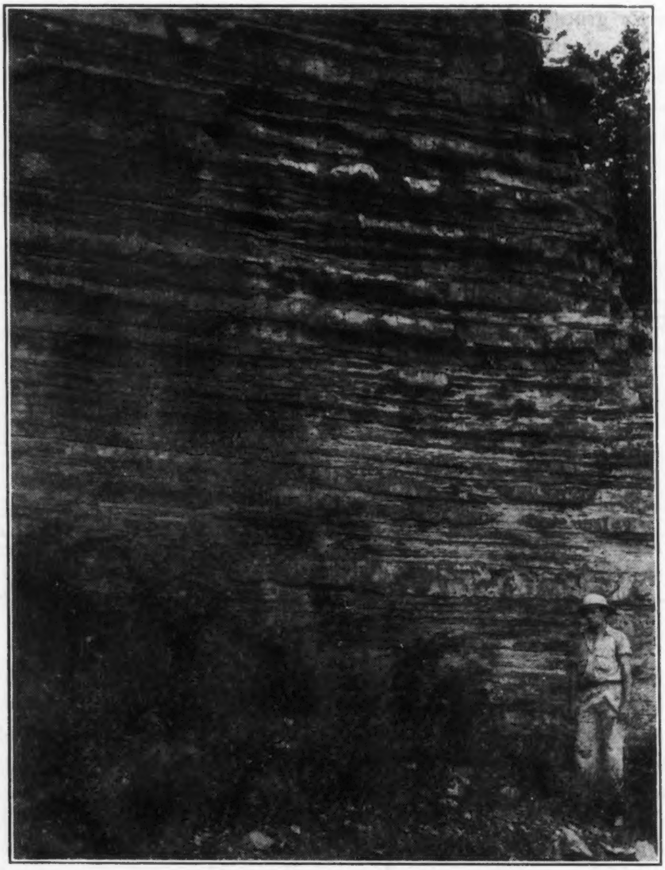


Fig. 29.—Typical Fort Payne Limestone and Chert

The larger solution channel described by Mr. Hays has formed along several large parallel strike joints. Because of the brittle character of the bedrock and a slight change in the dip of the strata, the rock had been literally torn apart to resemble a fault breccia at several places. The large tension joints and fractures disappear completely at the bottom of the solution zone on one of the more prominent bedding planes. If the joints had continued to a greater depth, so would have the solution. Thus, with a slight change in the dip of a uniform, brittle, cherty limestone, closely spaced joints can produce a major solution zone which perhaps is just as serious as a major fault.

During early exploration work considerable emphasis was placed on the superior value of angled core drill holes as compared with vertical holes, since the angle holes intercepted more of the vertical solution channels and they gave the width of the channels as well. These advantages are obvious and not too many angle holes were drilled, but, if more deep vertical holes had been completed first, several large continuous bedding plane seams would not have been overlooked. Even after the deviation of the angle holes had been measured by suspending hydrofluoric acid bottles in the holes and by measuring the angle with a transit sighted on a lowered flashlight in the hole, the deep holes curved so that both methods were inaccurate. As a result, the holes were never plotted correctly. Thus, the nearly horizontal seams were never recognizable from the staggered holes and they were not understood until construction was well under way. More deep vertical holes would have disclosed the bedding plane seams. The large solution channel described by Mr. Hays could have been recognized as easily from vertical holes since the high dips in the cores indicated tilted blocks, but a few angle holes would have given more information on the width of the channel.

In addition to the method described by Mr. Hays, this solution zone could have been treated by at least one other procedure. If the top of rock between the lock walls downstream from the lower gates had been cleaned off and a slab of concrete placed over the dissolved area, the solution channel could have been grouted extensively with clay where the cutoff was constructed. The concrete slab would have served to prevent too many grout leaks, and it would have prevented the washing out of the clay filling in the channel. With these precautions, and with the rather wide, deep cover of overburden upstream from the cutoff, the risk of possible leakage does not appear great.

Kentucky Dam[9] demonstrates the difficulties often encountered in limestone foundations that have long been exposed to intense weathering. A few closely spaced joints can make a tremendous difference. Even the most thorough exploration programs often leave much to be desired in cavernous limestones.

Mr. Schmidt's detailed account of the methods and equipment used to cut off the flow under Hales Bar Dam is a worthy contribution to engineering literature, but a more complete discussion of geologic conditions causing the cavitation would have been a welcome addition for a better understanding of the problems, since former literature on the foundation was similarly incomplete. The large amount of recent drilling should have afforded a much better picture of the geologic conditions than had been possible previously.

The "washing out of the shales in horizontal bedding planes, causing horizontal openings" is somewhat unusual and needs additional explanation. Shales are generally impervious and it is difficult to understand how the shales could wash out unless they were badly weathered or exposed in a cavity.

The photograph, Fig. 12(*a*), shows a considerable amount of cavitation following the bedding planes, but in Fig. 13 only a small amount of cavitation appears along the bedding of the limestone. This seems to be unusual and

[9] "Foundation Exploration at Kentucky Dam Site," by A. V. Lynn and R. F. Rhoades, *Engineering News-Record*, Vol. 125, 1940, pp. 70–73.

worthy of additional explanation. At Kentucky Dam nearly horizontal bedding plane seams, of from 0.1 ft to 0.5 ft thick, have been recognized for 4,000 ft down the slight dip.

In Fig. 13 both cavernous areas indicate a dip to the right approximately parallel to the reverse faults shown. Both cavernous zones seem to have developed in this pattern as a result of more general minor faulting and fracturing than is shown in Fig. 13. A probable minor fault could be assumed in Fig. 13 from Station 22+10, at the top of rock, to Station 19+85, at El. 485, where it becomes tight or dies out into bedding planes. Cavities do not need to exist everywhere along a fault, as the solid areas of rocks between the cavities represent pillars and there may be cavities only a few feet away behind these pillars. Once water has gained access to the limestone and a cavity has started to develop, solution of the limestone should take place in some part of the cavity away from the original fracture faster than along the opening.

If the cavernous areas are not the result of faulting and fracturing, why has not the entire river bed been affected uniformly since the limestone is essentially the same? Solution normally could not work its way below beds of impervious flat-lying shales unless they were broken. Small faults of only a few inches in limestones may afford just as good an opening to start solution as would a large fault. As shown in Fig. 13, curved fault planes are apt to produce additional small faults and fractures, which would aid solution. Indirectly, defect 3 (faulting) listed by Mr. Schmidt appears to have been the cause of both large cavernous areas which comprise at least 90% of the problem.

Geologic cross sections, such as those in Fig. 13, form an invaluable record, and, if a cross section had been prepared before construction of the dam, the seriousness of the problem would perhaps have been apparent. The problems of limestone foundations are not so difficult to treat when they are fully understood. Several errors seem to have been made at Hales Bar before the large leakage was finally cut off under Mr. Schmidt's direction. The first error was in misunderstanding the foundation problems or in not acknowledging their seriousness. The second error was in not having thoroughly grouted the rock for a considerable depth below the cofferdams before attempting to unwater.

Asphalt seems to have been used successfully with neat-cement grout, but the suitability of asphalt as a permanent material seems questionable. Mr. Schmidt noted that cavities frequently occurred above or around the former asphalt filling. He concluded that sand and clay had since been washed out to leave open cavities around the asphalt. Although this may be true in part, there are two other logical possibilities: (1) Gradual deformation and flow of the asphalt under the reservoir pressure, or slumping under its own weight if the cavities are not completely filled for some distance up and downstream, and (2) solution of the limestone since the asphalt was injected.

A perfect sphere of asphalt or pitch will flatten noticeably in one day under its own weight. In the course of several days the spheroidal shape of the asphalt is no longer noticeable. Columbus is supposed to have patched the leaks in his boats from the large asphalt lakes on the southwest corner of Trinidad; and vast quantities of the asphalt have been removed since Columbus visited the island, but the level of the asphalt remains approximately the same.

Asphalt is a semi-solid and its satisfactory behavior in cavities over a long period is doubtful. It is granted that a complete cutoff may be obtained in a large cavity without filling the cavity laterally for any great distance from the cutoff line. However, if the unfilled cavity has a slight downstream slope, there seems to be no reason why the asphalt seal will not be broken by slumping or slow flowing as a result of its own weight or the reservoir pressure.

If the asphalt is backed up with neat-cement grout as was done at Hales Bar Dam, this could be prevented. For the foregoing reason, up to 1940 the gradual increase in the leakage following asphalt grouting may have been due in part to a slow deformation of the asphalt.

During the construction of Watts Bar cofferdam, a large navigation dike made of limestone boulders was removed from the river channel. The dike had been constructed in 1912 by the Corps of Engineers, U. S. Army, and was removed in 1940, 28 years later. Most of the limestone boulders had been continuously under water and freely washed by the waters of the Tennessee River. After 28 years of exposure to solution the layers of limestone had receded by solution from one half to three fourths of an inch, and impure and cherty layers protruded the same distance. This example gives a very good idea of the normal rate of solution of a limestone in a free-flowing channel. This particular limestone was more dense and much tougher than the type occurring at Hales Bar Dam. Exposed to such rapid flow currents as existed under Hales Bar Dam, each wall of the solution channels might recede as fast as 1 in. in 25 years, or a narrow fissure only 1 in. wide might increase to 3 in. in width in this time. This factor may have contributed a noticeable amount to solution in the areas of broken rock and along the joints. For this reason a perfect cutoff is necessary in a limestone foundation.

The difficulties experienced with this and other dams constructed on limestone may cause many engineers to be dubious of a dam site on limestone. Regardless of this bad publicity, limestone foundations can be among the best if proper care is used in the selection of the site. Limestones have many good features not found in other rocks. They are usually soft and easy to drill, blast, and crush. Most limestones which have been subjected to great stresses have yielded by gliding within the calcite crystals; and fractures, shear planes, and joints are healed with cementation of calcite so that less difficulty is experienced than with many other rocks.

J. K. Black,[10] M. Am. Soc. C. E.[10a]—Complete and interesting descriptions of the cutoff programs at Kentucky and Hales Bar dams and records of procedures and results are contained in the papers of this Symposium. Both the problems and their solutions are exceptional even in a limestone formation where unusual conditions are expected. It is fortunate that the two papers were grouped as they were because the conditions which were corrected by Mr. Schmidt at Hales Bar Dam are a good illustration of what could have happened at Kentucky Dam if a cutoff by grouting alone had been considered acceptable.

[10] Project Mgr., TVA, Fort Loudoun Dam, Lenoir City, Tenn.

[10a] Received by the Secretary March 10, 1944.

Both papers illustrate the erratic changes that can occur in limestone formations, and emphasize the necessity for a great amount of exploratory drilling to secure complete information. They also demonstrate the need for, and the aid furnished by, the geologist in interpreting data, preparing drawings, and otherwise collaborating with the engineer on conditions and possible treatment of various formations.

A positive cutoff, in a badly weathered formation, at great depths below the water table, where rapid movement of large volumes of water occurs, is difficult to construct and requires the utmost study, skill, and perseverance. In addition to the difficulties encountered in handling the water, the engineer is confronted with the decision as to how far to go, especially in depth, in treating badly weathered formations under such extremely bad conditions. That decision is often very difficult to make.

At both Kentucky Dam and Hales Bar Dam, conditions were investigated thoroughly and completely and the objectionable features were treated and corrected in the same thorough and complete manner. There appears to be no doubt that the treatment was effective at both places. The methods adopted, the procedures followed, and the equipment selected evidence the thought and consideration that assured efficient execution and successful completion. The programs at both projects are to be commended.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FAILURE OF THE TACOMA NARROWS BRIDGE

REPORT OF THE SPECIAL COMMITTEE OF THE BOARD OF DIRECTION

Discussion

By Francis P. Witmer

Francis P. Witmer,[2] M. Am. Soc. C. E.[2a]—The various reports on the Tacoma Narrows Bridge collapse abstracted by the Special Committee of Board of Direction of the Society are very interesting, but not at all surprising. For many, if not for most, engineers they probably confirm preconceived conclusions. Almost a year to the day before the failure, at a meeting of the Philadelphia Section of the Society devoted to a discussion of the then newly completed Bronx-Whitestone Bridge, the writer presented a list of important suspension bridges, with their ratios of span length to depth of stiffening girder. This list included many of the bridges in Table 1. The writer emphasized the extreme ratio (209) used for the Bronx-Whitestone Bridge which had exhibited undesirable characteristics in service, and expressed the opinion that this ratio was probably too high for this structure, as compared with the ratio which had proved wholly satisfactory in other bridges. The Tacoma Bridge was then being designed and, when its proposed ratio of 350 was mentioned by a speaker at the meeting, the writer emphatically expressed his disapproval of it.

With the exception of Report (*e*), all the published reports coincide in the conclusion that insufficient depth of girder with relation to span length was a vital factor in the collapse. This should not be considered as other than a serious defect in the design. The extreme lack of torsional resistance, an obvious fault, is also cited in the reports.

Report (*a*) states that "'The Tacoma Narrows Bridge was well designed and built to resist safely all static forces, including wind, usually considered in the design of similar structures.'" This statement is open to the criticism that there is no similar structure with such an extreme span-to-depth ratio, combined

Note.—This report was published in December, 1943, *Proceedings*.

[2] Prospect Park, Pa.

[2a] Received by the Secretary March 21, 1944.

with such relative lightness of construction. The statement is also made: "'It was not realized that the aerodynamic forces which had proven disastrous in the past to much lighter and shorter flexible suspension bridges would affect a structure of such magnitude as the Tacoma Narrows Bridge, * * *.'" This does not offer an adequate justification for the use of such extreme features in design.

Report (*a*) further states that "'there is no doubt that sufficient knowledge and experience exist to permit the safe design of a suspension bridge of any practicable span.'" With this the writer unreservedly agrees. The only reason for the departure from tried and successful proportions in this instance seems to have been the desire for greater economy. Under the heading, "General Features of the Bridge Project: Financing," the statement is made that "it was considered necessary to keep the cost of construction as low as possible." A similar motive underlay the design of the ill-fated first Quebec (Canada) Bridge. Neither for that structure nor for the Tacoma Narrows Bridge will the total ultimate cost be any lower because of the effort to economize. No reasons of either economy or esthetic appearance should control the design of a structure intended for public traffic. If a radical departure from established successful practice is contemplated, to save expense or for any other reason, preliminary study first should demonstrate that the resulting structure will not only possess the necessary qualities of strength, but will also be free from any alarming or disquieting performance. The misbehavior of the Bronx-Whitestone Bridge and of the bridges at Thousand Islands, Deer Isle, and elsewhere, all attributable to similar causes, was a matter of general engineering knowledge and should have served as a warning against the dangers of so experimental a design.

The Special Committee of the Board of Direction requests that the investigation of the safety of suspension bridges and their adequate design under aerodynamic forces be initiated and undertaken by the Public Roads Administration. Interesting as such a study would undoubtedly be, the design of structures of this character, for a considerable time in the future, will probably be governed by established conservative practice, rather than by modernized methods requiring an application of the principles of aerodynamics.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DEVELOPMENT OF THE CHICAGO TYPE BASCULE BRIDGE

Discussion

BY DONALD N. BECKER

DONALD N. BECKER,[12] M. AM. SOC. C. E.[12a]—Foundation conditions, especially the unreliability of some borings, are discussed by Mr. Hammond. For that reason borings made in later years have been continued 10 ft into rock to avoid the possibility that the first report of "rock" was merely a boulder. The accumulated mass of data that has been gathered through the years also tends to verify the results. Rock along the river in the downtown district is usually encountered at elevations slightly more than 100 ft below Chicago Datum (normal lake level) from Lake Michigan to the Lake Street bridge. The rock elevation then rises gradually southward with rock at about El. − 60 at the Roosevelt Road bridge and at about El. − 25 near 14th Street. Rock occurs at lower levels again south of 14th Street but not as deep as at the Lake Street bridge.

The rock is generally overlain with a fine water-bearing sand of varying thickness, covered by a hard clay layer of considerable thickness—say, as thick as 20 ft or 25 ft—which gradually softens as the elevation rises so that soft clay is reached usually above El. − 40.

The presence (in some cases, of as much as 15 ft or 20 ft) of water-bearing sand just above rock has made penetration to rock difficult and expensive. In some such cases subpiers have been belled out on hardpan after a test boring has been made to see that this layer is at least 15 ft thick. The belling out has usually extended until the unit pressure is reduced to 8 tons per sq ft. This practice has been found desirable in only a few cases in each of which rock was at, or more than, 100 ft below datum. No difficulties from settlement have been encountered.

Mr. Hanover presents a novel solution of the skew crossing problem, but the writer wishes to call attention to the relatively narrow channel in this case.

NOTE.—This paper by Donald N. Becker, M. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by Alonzo J. Hammond, Past-President and Hon. M. Am. Soc. C. E.; September, 1943, by Messrs. Clinton D. Hanover, Jr., and Armour T. Granger; and January, 1944, by C. B. McCullough.

[12] Engr. of Bridge Design, Dept. of Public Works, City of Chicago, Chicago, Ill.

[12a] Received by the Secretary March 1, 1944.

Even if a similar leaf were placed on the other side of the channel to provide a wider opening, there would have been a channel of less than 100 ft. The bridges that Mr. Hanover refers to in Table 1 are rather old and in remote or narrow parts of the river system. In general, no channels of less than 140 ft are now permitted, and preferably channels should have from 170 ft to 200 ft clear span. With such great widths the Hamilton Avenue type bridge would not be practicable, especially where semithrough trusses are indicated instead of the deck girders at Hamilton Avenue.

In March, 1940, a bridge design was being developed for the proposed Congress Street superhighway where the skew is 62° 29′ and a 170-ft clear channel was required; the grade was high enough to permit deck trusses. As the proposed bridge was quite wide, 109 ft over-all, to provide two 44-ft roadways separated by a 3-ft center dividing fin and two 9-ft sidewalks, a square double-leaf bridge would have required from about 245 ft to 250 ft between masonry. A layout was finally developed in which two bridges will be placed side by side with a longitudinal joint in the center fin. In this layout one bridge will be set 31.61 ft ahead of the other. Thus, the clear span between masonry was reduced to 214.70 ft. These two bridges will be operated simultaneously by one operator on each side of the river as in the present Chicago double-leaf bridges. Arrangements will be made so that either bridge may be operated independently, which will permit one bridge to be placed up in the air for repairs and yet allow traffic on the superhighway to continue by using the other bridge for traffic in both directions on its 44-ft roadway until the repairs are completed.

Before this arrangement was developed, a study was made in which trusses were placed at right angles to the channel, with part of the outer truss in the acute angle of each leaf entirely outside of the bridge sidewalk. Since this plan produced a layout of questionable appearance, another layout was prepared in which the trusses were bent in a vertical plane with the trusses in front of the masonry parallel to the street but with the rear ends of the trusses normal to the channel. This layout was not considered desirable because of the difficulty of providing for the large kick reaction at the bend in the truss and the unequal power requirements on operating racks.

Mr. Hanover's remarks on the use of a hydraulic drive such as that adopted for the Canal Street bridge are not without merit. However, this installation is an attempt to develop a drive that will eliminate the complicated and expensive electrical control system of the present electric drives. It is being tried out as a transition from the usual electric drive to full hydraulic drive, and will have an auxiliary electric motor for emergency use. However, this electric motor drive will not have the complicated and expensive magnetic control equipment of the usual electric motor drives ordinarily specified in Chicago but will be controlled from a simple drum-type controller. If, after a few years of trial, the hydraulic drive proves reliable, future bridges may be equipped with it alone.

Chicago engineers also encountered Mr. Hanover's difficulties with control devices for the hydraulic drive and had to originate their own. A controller has been developed which operates the same as does the electric controller with which all the operators are familiar, in which the handle moves in a

vertical plane—a pull on the handle toward the operator from a vertical position causes the leaf to rise and a push forward from the vertical causes the leaf to lower. The handle on the hydraulic drive is similarly arranged, and the greater the deflection of the handle is from the vertical the greater the speed will be. One great advantage of the hydraulic drive over the electric drive is that a return of the handle to the vertical automatically stops the leaf in its position without the application of the brake. A brake is provided, however, for emergency holding and for use with the auxiliary electric drive.

Professor Granger comments on the desirability of the simple trunnion bascule of the Chicago type. Its main advantages are: Freedom from linkages with pins that wear, and which may bind, in case of failure of lubrication; freedom from the hinged traps sometimes required with the rolling lift type; and other similar difficulties. Professor Granger is correct in his assumption that lack of space limited the discussion of details. However, these are so often affected by the requirements of each site that they can seldom be applied at another location and any experienced bridge engineer can easily develop details to suit his location. The main purpose of the writer was to chronicle the history and development of the Chicago type—the most significant phase of which was probably in support of trunnion bearings.

Professor Granger states that "A thoroughly satisfactory lock should grip both leaves firmly, without play or clearance, * * *." As against this need one should not lose sight of the necessity that the horizontal movement should allow for the temperature expansion of each leaf. It is not desirable to place one leaf on rollers so it can move in case of temperature expansion. (This was done in the case of the Soo Canal bridge where a full moment lock was provided to make the entire bridge function as a simple span under live loading. A failure of this lock for some undetermined reason was the cause of dropping one leaf of this bridge into the Canal on October 7, 1941.)

Locks on the Chicago bridges now consist of I-shaped bolts operated by electric-driven gear trains that force the bolts into the sockets on the other leaf. The sockets are provided with suitable adjustment features to allow for wear. The bolts are so designed that in case of necessity the leaves may be operated and the bolts can be withdrawn without damage to either bolts or sockets. Latest designs provide for the operation of these bolts by means of hydraulic pistons instead of motor-driven gear trains. Chicago's bridges have had their full share of difficulties with center locks.

Since before 1924, new bridges and some of the older ones have been equipped with intermeshing finger castings at the center roadway break. These do not foul each other when they mesh because the clearance tolerance of the teeth is greater than the clearance in the side cheeks of the center-lock castings which aline the bridge leaves before the teeth come opposite each other.

The writer admits that the general use of the Pratt type of trussing has never been given much consideration beyond the thought that with a curved bottom or top chord the difference in slope of the diagonals on each side of a vertical does not look well for Warren type trussing. A better appearance is obtained with the Warren type if parallel chords with attendant equal slopes are used. Also, the angle between the diagonals that were reversed and the

sloping bottom chord would be so small that connection details would be difficult.

In regard to the use of a live-load bearing forward of the trunnion, several considerations have led Chicago engineers to this practice. First, the necessary supporting girders, whether of the S-type or cross-girder type, would otherwise have to carry the live load on the bridge and these girders are already of very heavy design. Second, in case of maintenance work the presence of the live-load support has provided a ready means of relieving the dead load from the trunnions by simply jacking up on the rear end. One such example was described by S. J. Michuda in 1942.[13] In this case because of a bump the entire east leaf of the East 106th Street bridge had shifted eastward and, by methods described in that article, the bridge was brought back to proper position and reanchored.

Unequal deflection between the supports for the inner and outer trunnion bearings is compensated by calculating the anticipated deflection of the S-girder and by setting the inner trunnion bearing higher. No ill effects have been experienced. Professor Granger comments on transverse centering and indicates that he has found no need for such devices. In Chicago bridges where the span is long enough to require trusses, designers usually provide cheeks on the center lock which guide the bridge into correct alinement within the allowed tolerances. Of course, where the end of the leaf is away from the trunnion 100 ft to 125 ft, only a relatively small force is necessary to deflect the leaves.

The writer agrees with Professor Granger on the merits of vertical lift versus bascule bridges. When a lift bridge can be placed out in a clear open location as was the Triborough Bridge in New York, it is definitely desirable. In the past too little attention has been given to the appearance of a structure and too much stress to the lowest possible cost, with the result that many monstrosities have been foisted upon the long suffering public. The writer believes that day is gone.

Mr. McCullough's comments are welcomed as helping to round out the entire topic by summarizing the numerous advantages of the bascule type bridge over other types and by outlining the necessity for, and the manner of, providing interlocking for the various operations.

Chicago's new bridges are thoroughly interlocked but by-passes must be provided so that a failure at any point will not stop the entire functioning. These by-passes are sometimes the cause of trouble if used inadvertently or otherwise when not actually needed, thus defeating the very purpose of interlocking. In this way one small human failure may defeat the best efforts of science.

Corrections for *Transactions:* In February, 1943, *Proceedings*, page 264, Fig. 1, sites 34 and 6 coincide; on page 265 change site number 39 to 52; on page 266, line 3, delete "39, Cortland Street and North Branch"; on page 280, line 42, change "hard clay" to "rock"; and, on page 284, line 6, change "16 lb per sq in." to "16 lb per sq ft."

[13] "Dislocated Bascule Rocked Into Position," by S. J. Michuda, *Engineering News-Record*, April 9, 1942, p. 85.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA

Discussion

BY LUNA B. LEOPOLD

LUNA B. LEOPOLD,[31] JUN. AM. SOC. C. E.[31a]—When the writer started the preparation of the tables of frequency-magnitude of precipitation (Table 3) for nonrecording rain gage stations in New Mexico and Arizona, he was confronted with the task of making estimates of design floods for numerous flood control structures on small watersheds scattered throughout the two states. Each time such a design problem arose the rainfall records available for stations near the watershed in question had to be analyzed. The watersheds were scattered through two states where topographic relief, elevation, soils, and rainfall vary tremendously within short horizontal distances, and the flood control structures were often to be located in inaccessible country where rainfall stations were scarce. It was necessary, therefore, not only to analyze rainfall records for stations adjacent to the watershed in question but to attempt to find other watersheds where the factors affecting runoff would be comparable. Thus, tabulations of rainfall records soon were accumulated for rainfall stations located all over the two states. In practically all cases, intensity records were lacking for stations where rain analyses were necessary.

As a result of these circumstances and limitations of time, the writer was not able to make detailed studies of storms of long duration which would be applicable to the design of major structures on large watersheds. However, even the problem of intensities and frequency of storms of small areal extent in the Southwest has always been one of considerable importance owing to the scarcity of rain gages, particularly intensity gages, in such a large area of diverse topography. The writer wishes to emphasize the comments made by Mr. Girand and Mr. Hodges that, as stated in the paper, the frequency data were prepared primarily to study the storm rainfall on small watersheds, and

NOTE.—This paper by Luna B. Leopold was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by Lawrence Pratt; June, 1943, by L. L. Harrold and A. J. Dickson, and James Girand; November, 1943, by Clarence S. Jarvis; December, 1943, by Paul V. Hodges, Edgar E. Foster, R. W. Davenport, and A. K. Showalter; and January, 1944, by Walter B. Langbein, and Emil P. Schuleen.

[31] 2d Lt., Air Force, U. S. Army, Los Angeles, Calif.

[31a] Received by the Secretary February 21, 1944.

have limited applicability to long duration storms. Mr. Schuleen suggested that analyses of the rainfall for long duration storms would add to the value of the paper. The writer is in entire agreement and regrets that the study could not, at that time, have been so extended.

The data presented under the heading, "Comparison of Magnitude of 1-Day, 2-Day, and 3-Day Rains," and discussed by Mr. Jarvis represent a preliminary approach to the problem of long duration storms, but, as stated in that section of the paper, the results are indicative only. Even since the installation of additional intensity gages by the Weather Bureau in 1940, numerous storms of long duration and considerable magnitude have occurred, the records of which should certainly be considered in any detailed study of extended storms in the area.

The variability of the frequency-magnitude values for closely adjacent stations was noted by Messrs. Girand, Hodges, and Schuleen. At first glance, these variations seem to invalidate the final results, and they lead Mr. Schuleen to conclude that either the basic data are inconsistent or the method of analysis is at fault. The writer called attention to these variations under the heading, "Isopluvial Maps," and, to give added prominence to the variations, the 50-yr values were plotted on topographic maps in Figs. 3 and 4.

This variation between stations is a reflection of an important characteristic of the summer-type rainfall. Because of the physical nature of the short duration storms, the area of most intense rainfall is small, and one station may receive a large amount of rainfall while adjacent stations receive none. An example is the storm at Las Cruces, N. Mex., on August 29, 1935. At that station 6.49 in. of rain fell in 7 hr while other stations in the vicinity recorded "none" or "trace" up to 1.83 in.[32]

Both from the standpoint of the actual frequency figures computed and from a consideration of the physical character of the short duration storms, the element of chance seems to play an important part in determining the rainfall values actually recorded at a given station. The available records for the Southwest generally cover too short a period to give either a true picture of the frequency of the infrequent storms, or consistent frequency values at any pair of adjacent stations.

W. V. Turnage and T. D. Mallery[33] analyzed one year of rainfall records from sixteen standard rain gages in a 300-m square plot and concluded that, although one rainstorm gave considerable variation of precipitation between gages, at the end of the season, the total rainfall for each gage compared closely with the average seasonal rainfall for all gages. They noted that the variation in total seasonal rainfall between near-by gages was smaller for stations in Arizona near Tucson than for stations near Yuma, and concluded that spatial variation in seasonal precipitation is greater in areas of low rainfall than in areas of high rainfall.

The variation between adjacent stations does tend to iron out as the length of record increases, but, when a station happens to experience an exceptionally

---

[32] "Areal Extent of Intense Rainfall, New Mexico and Arizona," by Luna Leopold, *Transactions*, Am. Geophysical Union, Pt. II, November, 1942, pp. 558–563.

[33] "An Analysis of Rainfall in the Sonoran Desert and Adjacent Territory," by W. V. Turnage and T. D. Mallery, *Publication No. 529*, Carnegie Inst. of Washington, 1941.

great rainfall, a long time might elapse before other rains of that magnitude are recorded again at the station (as, for example, at Las Cruces). In the 48-yr record at Las Cruces the greatest rain recorded in 24 hr was 6.49 in., which actually fell during a 7-hr period. The second largest rain at that station was 2.43 in.

The topographic maps of Figs. 3 and 4 show that the frequency-magnitude values in a given broad area differ considerably from the data for other areas in a different geographic setting, in spite of the tendency for variations between adjacent stations to mask the broad differences. As an example, compare the data for a 50-yr expectancy (summer) for stations in a west-east strip across east central New Mexico. The stations chosen for Table 6 were divided arbitrarily into three groups separating stations into geographic areas more or less

TABLE 6.—COMPARISON OF 50-YR (SUMMER) FREQUENCY-MAGNITUDE VALUES FOR STATIONS IN EAST CENTRAL NEW MEXICO

West			Central			East		
Station	El. (ft)	50-yr frequency (in.)	Station	El. (ft)	50-yr frequency (in.)	Station	El. (ft)	50-yr frequency (in.)
Stanley	6,317	3.09	Palma	7,000	5.40	Logan	3,851	3.80
McIntosh	6,355	3.25	Cuervo	4,849	3.06	Tucumcari	4,100	3.58
Estancia	6,300	2.90	Santa Rosa	4,624	2.35	San Jon	3,982	4.55
Tajique	7,100	3.45	Pastura	5,285	3.00	Montoya	4,335	5.60
Mountainair	6,475	3.11	Vaughn	5,930	3.40	Melrose	4,400	4.25
....	....	....	Lagunita	4,500	4.30	St. Vrain	4,250	3.50
....	....	....	Alamogordo	4,338	3.65	Clovis	4,262	5.01
....	....	....	Ft. Sumner	4,028	3.85	....	....	....

centered around Torrance, Guadalupe, and Quay counties. In this strip of territory there are no important mountain ranges, and all stations are approximately at the same elevation. However, the stations in the Estancia Valley area (Torrance County) are more or less protected by the Sangre de Cristo Mountains to the north, the Manzanos on the west, and the Gallina-Capitan ranges on the south. This relative isolation differentiates the Estancia Valley from the high plains along the east edge of the state where the effect of the frontal weather of west central Texas is often felt. No important topographic barriers protect the high plains stations from southerly wind currents.

Table 6 indicates an increase in the magnitude of 24-hr summer-type rain of 50-yr frequency from Torrance County on the west to Quay County on the east, although closely adjacent stations vary considerably between themselves.

The manner of grouping stations or records that do so differ is a question on which probably no two persons would agree. One method might be by station-year grouping. The requirements for the use of such a grouping were discussed under the section, "Applicability of Station-Year Analysis." As stated there, frequency curves differ so much that it is difficult to decide whether the rainfall experience at certain stations differs mostly due to chance, or whether assignable causes, such as topography, were operative. The assignable causes may operate in subtle ways which are not immediately apparent. Thus, different persons may not agree on which stations should be grouped together.

In order not to generalize the basic data by grouping stations, the writer presented the frequency data separately for each station, and then generalized in the construction of the isopluvial map. Since the number of rain gages is small per unit area, the short records are included even though their value is less than that of the long records. For comparability the short records were analyzed in the same manner as longer records, and the engineer who uses the data must consider the length of record in weighing its worth. To facilitate this, the writer included in Table 3 the exact years of record used in making the frequency arrays. The writer agrees with Mr. Schuleen that the 100-yr frequency value based on a 15-yr record is of questionable reliability, but the short records should be considered rather than disregarded. Since the grouping of stations appeared to be open to many questions, particularly concerning which stations should be thrown together, the writer considered it justifiable to present the analysis of the short records as well as that of the long records, with the caution that reliability of the computed frequencies varies.

The use of the highest rain on record each year for the construction of frequency curves is comparable to the annual flood method.[34] The recurrence-interval or California Method of plotting[35] has had considerable popularity, but it represents only one of many procedures which might have been employed in this analysis and, as Mr. Langbein stated, gives values somewhat higher than other methods.

Comparison of the results obtained by various methods of plotting indicated that the magnitude of rainfall values for frequencies of 50 to 100 years was determined mostly by the four or five highest rainfall values recorded at the station. If these highest rainfall values fall on a smooth curve, the method of computing the plotting position is of importance. However, for many stations whose records were short, the highest values did not plot in a smooth curve and the fitting of a curve through the points was a matter of individual judgment. Consequently, the writer concluded that the different methods of plotting in such a case give less variation in final values than would the positions of a smooth curve drawn through the plotted points by different workers.

The construction of an isopluvial map such as Fig. 5 is open to many kinds of errors, as Messrs. Langbein and Davenport have explained. It is presented, however, as one way the frequency data may be generalized to give a rough over-all picture of the geographic distribution of the rainfall expectancy. As shown by Mr. Davenport, with a large number of 50-yr samples of maximum 24-hr rainfalls, these maxima would be scattered around their mean in a probability curve, or with random distribution. This would be true if there were no assignable causes such as topography to make the rainfall differ between stations. In such a case, the geographic distribution of maxima would also be random.

However, when the data are plotted on a map, the geographic distribution of values is not random but, as demonstrated by Fig. 5, tends to show a distribution which can logically be explained by topographic differences and relation of the area to sources of moisture. Thus, if an isopluvial map were properly

[34] "Floods in the United States," by C. S. Jarvis and others, *Water-Supply Paper No. 771*, U. S. Geological Survey 1936, p. 54

[35] *Ibid.*, p. 58.

analyzed, it should provide a key to the geographic distribution of causes which tend to vary the record between stations—all causes being integrated into the final map. In practice, however, only general principles can be cited to explain why isopluves are distributed as they are. More detailed studies of macro-climate and micro-climate are necessary before all the obscure effects of assignable causes can be evaluated.

The rainfall frequency for a given small area should be derived from the applicable station frequency-magnitude curves rather than from a general isopluvial map as presented in Fig. 5. As a generalized picture of the distribution of frequency data over a large area, the isopluvial map is a convenient picture but should not be trusted alone to define the frequency-magnitude values for a given spot.

Mr. Langbein points out that the highest 24-hr isopluve for 50 years shown by Mr. Yarnell[2] in Arizona is 3.5 in., whereas the writer placed a 6-in. isopluve over the mountain area surrounding Crown King, Ariz. This difference might be expected inasmuch as the maps constructed by Mr. Yarnell were based on only two intensity records in Arizona—at Phoenix and Flagstaff. In the record analyzed by Mr. Yarnell, the greatest rain at Phoenix was 1.15 in. in a 30-min period, whereas the greatest 24-hr rain (during the summer season) in the writer's analysis for Phoenix was 2.91 in. In a 25-yr record, on the other hand, Crown King experienced three rains in the summer season which exceeded 3 in. in 24 hr, the largest of which was 5.05 in.

Thus, the importance of adding to the very valuable material published by Mr. Yarnell is apparent, even if the data available for analysis are subject to the limitation that they are 24-hr totals and not intensity records.

Although the isopluvial values presented by the writer in Fig. 5 are in closer agreement with those of Mr. Yarnell for New Mexico than for Arizona, they are higher for both states. The isopluves of Mr. Yarnell for New Mexico agree very satisfactorily with those derived by Robert L. Lowry, Jr.,[36] M. Am. Soc. C. E., for the western part of Texas. A reconsideration of the position of the isopluves in Fig. 5 for eastern New Mexico would probably be profitable in order that they might be drawn in a more easterly position for better agreement with those of Messrs. Yarnell and Lowry.

Mr. Langbein's valuable discussion presents more detailed studies of the relation of intensity to elevation. The writer indicated that the center of greatest precipitation occurred near the base of the mountain slope. Mr. Langbein's further analysis of the same data showed a decrease of intensity with elevation. The same result was reached by Mr. Brancato[29] whose conclusions were supported by Mr. Showalter at least from theoretical considerations; but Messrs. Harrold and Dickson show that the data from Navajo Experiment Station are not in agreement with this conclusion. As Mr. Harrold stated, there is need for additional information before the details are understood.

[2] "Rainfall Intensity—Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U.S.D.A., August, 1935.

[36] "A Study of Rainfall in Texas," by Robert L. Lowry, *Bulletin No. 18*, State of Texas Reclamation Dept., August, 1929.

[29] "The Meteorological Behavior and Characteristics of Thunderstorms," by G. N. Brancato, Hydrometeorologic Section, U. S. Weather Bureau, April, 1942 (processed).

In summary, the lack of long records and the scarcity of rain gages make it necessary to utilize records that are really too short to be considered reliable for frequency determinations. The physical character of summer-type storms leads to differences between records of adjacent stations. These differences result from a combination of chance and assignable causes, which cannot be satisfactorily separated. Workers agree that most of the high rainfalls which fall in summer, and are recorded as 24-hr totals, actually fall in periods of less than 10 hr, and a study of intensity records and times of beginning and ending of rain shows that many of these rains fall during periods of 2 to 3 hr.

The intensity-time curves presented in Fig. 1 probably represent the intensity patterns for most of the summer-type, 24-hr rains which were analyzed for frequency, but, at best, the combination of an average intensity histogram with a total rainfall amount for a given frequency only provides additional data for depth-intensity-frequency relations of "point" rainfall. The relation of these factors to areal distribution of a single storm still remains to be demonstrated by research such as that being conducted at Navajo Experiment Station and Southwestern Forest and Range Experiment Station.

Engineers who use the data presented in Table 3 should do so with caution, because in many cases the variation of frequency values between adjacent stations cannot be explained satisfactorily. Only by detailed studies of assignable causes can the effect of chance be properly evaluated. Consideration of the frequency-magnitude curves for individual stations will lead to a better choice of a composite frequency value than will the use of a generalized isopluvial map.

Because of the inherent differences between long duration storms typical of winter and the intense thunderstorm type of summer rainfall, the frequency-magnitude values are applicable primarily to small watersheds, whose maximum flood flows result from the cloudburst type of storm.

The writer wishes to express his appreciation for the contributions made by the discussers. They are pertinent and valuable additions to the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MILITARY AIRFIELDS

A SYMPOSIUM

Discussion

BY W. E. HOWLAND, AND DAVID S. JENKINS

W. E. HOWLAND,[7] ASSOC. M. AM. SOC. C. E.[7a]—The main purposes of this discussion are (1) to present a mathematical method for the determination of a runoff hydrograph for a rain of limited duration, and (2) to compare the hydrograph for a particular rain and area so obtained with that presented by Mr. Hathaway. The same differential equation will be used as was developed by Mr. Horton for Eq. 2, which furnishes the basic curve used by Mr. Hathaway (18).[7b] Hydrographs have been plotted from the derived formulas and are shown in Fig. 27 for comparison with Fig. 18. Curves 3, 4, 5, and 6, Fig. 27, have been obtained mathematically by the writer's methods, the basic differential equation (18) being:

$$\sigma \, dT = K \, y^2 \, dT + \tfrac{2}{3} \, dy \qquad (7)$$

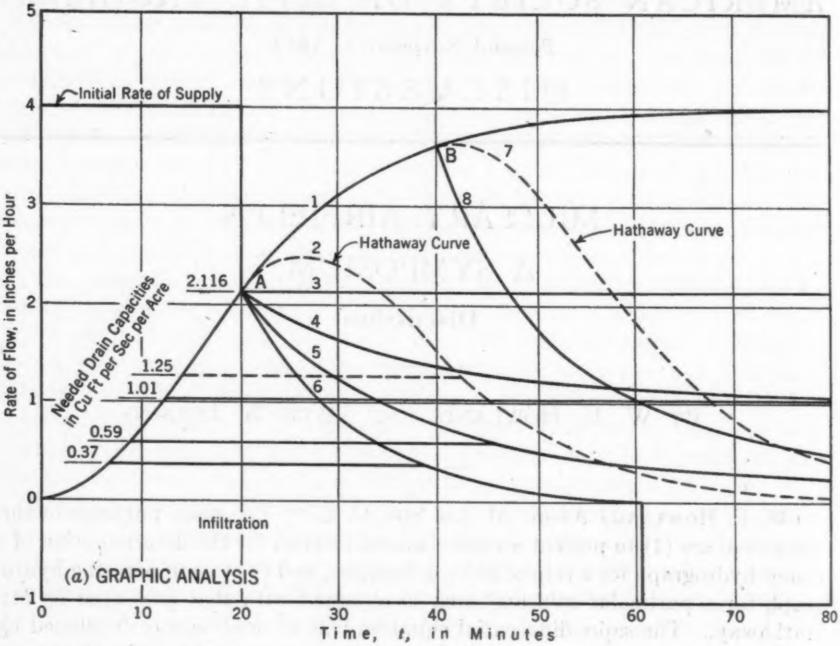
Each of these is intended to be the runoff hydrograph that follows a 20-min-long rate of supply, of 4 in. per hr intensity; but the assumption as to the magnitude of the rate of supply of the rain which follows the initial 4 in. per hr rate is different for each curve. In obtaining curve 3, Fig. 27, the subsequent rate of supply which lasts indefinitely was assumed to be 2.116 in. per hr. (This is a horizontal line.) For curve 4, the subsequent rate of supply of 1 in. per hr is assumed to last indefinitely; for curve 5, the subsequent rate of supply is assumed to be zero. (It might be supposed from reading Mr. Hathaway's explanation that curve 2, Fig. 18, was also based on this assumption of a subsequent zero rate of supply.) In obtaining curve 6, Fig. 27, it was assumed that the ground would absorb water at a rate of 1 in. per hr as long as there was water standing upon it. This assumption might be stated as a negative

NOTE.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; and March, 1944, by Thomas E. Stanton.

[7] Prof., School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

[7a] Received by the Secretary February 7, 1944.

[7b] Numerals in parentheses, thus: (18) refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.



(b) Mathematical Analysis

Curve No.	General equation[a]	Conditions[b] σ	Conditions[b] y_a[c]	Formula for given conditions
1[d]	$q = \sigma \tanh^2 1.5\, T \sqrt{\sigma K}$	4	0	$q = 4 \tanh^2 0.0461\, t$
2	By Hathaway method			By Hathaway method
3	....	2.116	....	$q = q_a = \sigma = 2.116$
4	$q = \sigma \coth^2 \left(1.5\, T \sqrt{\sigma K} + \coth^{-1} y_a \sqrt{\frac{K}{\sigma}}\right)$	1[e]	1.578	$q = \coth^2 (1.383\, T + 0.842)$
5	$q = \frac{K}{(1.5\, K\, T + 1/y_a)^2}$	0	1.578	$q = \frac{0.85}{(1.275\, T + 0.634)^2}$
6	$q = -\sigma \tan^2 \times \left(\tan^{-1} y_a \sqrt{-\frac{K}{\sigma}} - 1.5\, T \sqrt{-K\sigma}\right)$	−1	1.578	$q = \tan^2 (0.969 - 1.38\, T)$
7	By Hathaway method	....	....	By Hathaway method
8[d]	$q = \frac{K}{(1.5\, K\, T + 1/y_a)^2}$	0	2.062	$q = \frac{0.85}{(1.275\, T + 0.485)^2}$

[a] In addition to the notation of the Symposium: T = time, in hours; $t = 60\, T$ = time, in minutes; q and σ are in inches per hour; q_a = flow at the end of 20 min; y is in inches; y_a = depth at the end of 20 min; and K is in (hours-inches)$^{-1}$. [b] The following constant conditions apply to each curve in this example: $K = 0.85$ per hr-in.; $q_a = 2.116$ in. per hr. [c] Initial value. [d] For all curves except 1 and 8, T and t are measured from point A (t = 20 min). For curve 8, t and T are measured from point B (t = 40 min). [e] $\sigma < K\, y^2_a$.

Fig. 27.—Comparison of Runoff Rates and Drainage Requirements as Derived by Mathematical and Graphical Methods

rate of supply of 1 in. per hr for the subsequent period of ground submergence. This situation might conceivably result from a rain of 5 in. per hr lasting for the preliminary period of 20 min during which time, and also subsequently, the rate of infiltration was 1 in. per hr.

Curve 8, Fig. 27, was computed by the writer's methods for assumptions corresponding to those of curve 5, but with the runoff following a 40-min-long storm. Curve 7 has been constructed for comparison for this case using Mr. Hathaway's methods. The decrease in rate of runoff following the storm is even more abrupt in this case than for that of the 20-min storm.

The sharp break in the writer's hydrographs at the end of the period of high rainfall is the most conspicuous difference between Fig. 27 and Fig. 18. This characteristic has also been noted for comparable areas by others and is recorded in the literature (24)(25).

The effect of the different hydrographs on the estimate of needed drain capacity when storage is provided is also shown in Fig. 27. The same storage was assumed as in the example illustrated by Fig. 21 of the Symposium; and similar methods of analysis were employed. Using hydrograph 2, Fig. 27, the necessary drain capacity is 1.25 cu ft per sec per acre; but, with hydrograph 5, the needed capacity is only 0.59 cu ft per sec per acre; with hydrograph 4, the capacity is 1.01 cu ft per sec per acre; and, with hydrograph 6, the capacity is 0.37 cu ft per sec per acre. On the other hand, with hydrograph 3, the capacity becomes 2.116 cu ft per sec per acre. (The areas were measured in this analysis by a planimeter.) Thus, in this example the drain capacity requirement as determined from Fig. 18 seems about equivalent to that which might be needed for a rate of supply of 4 in. per hr lasting for 20 min, followed by a rain of somewhat more than 1 in. per hr with a rate of supply lasting indefinitely, or for a rain whose rate of supply is 4 in. per hr and lasts for 20 min, followed by another of about half that rate of supply which lasts for another 20 min and then diminishes to zero. Whether or not this example makes possible a true appraisal of the meaning of the results of Mr. Hathaway's methods would have to be determined by more extensive studies.

These curves and examples raise the question as to whether or not the curves used by Mr. Hathaway properly allow for the infiltration which certainly must continue after the storm ceases as long as the ground is submerged. The effect of subsequent infiltration is shown clearly by curve 6, Fig. 27.

Why does not the mathematical method yield precisely the same results as does the Hathaway method for a given intensity and duration of rain? To be specific—why do curves 2 and 5, Fig. 27, differ so markedly? The lack of agreement seems to rest on the failure of the unit hydrograph method employed by Mr. Hathaway to apply to the case. His method assumes that the rate of flow out of a given area which comes from a particular portion of a rainstorm is precisely the same at a given time after that rain occurred, regardless of the amount or distribution of rain that might have followed. Thus, according to this view, the flow rate out of an area from a 40-min-long rain of 4 in. per hr rate of supply would be the sum of two flow rates: (1) The flow rate that did occur during a like storm from this area 20 min after the beginning of that storm; and (2) the flow rate that would occur at a certain time from that area

as the result of a storm which lasted just 20 min but which ended 20 min before that time.

This assumption would be a valid one if at the end of the first 20 min an artificial roof, with hydraulic characteristics the same as those of the ground, were suddenly placed over the area. Under these conditions the two flows could not influence one another, and the Hathaway method of adding the two known independent flow rates would be correct.

The actual situation is different. The rain from the second 20 min of the storm finds itself, so to speak, flowing on top of the rain from the first 20 min of the storm. Each layer increases the velocity of the other. (Since the depths are greater at corresponding points on the area, and at corresponding times on this area, than in the hypothetical "double-deck" area, the velocities are greater.) One might first suppose that this increased velocity would not necessarily increase the rate of runoff, since the depths might decrease correspondingly, but, when the depths decrease, the amount of storage also decreases. As a consequence, a greater amount of runoff must have occurred in a given period of time.

From these considerations it appears that the mingling of the two flows, the one from the first 20 min and the other from the second 20 min of the rain, produces rates of runoff which, for a time, are greater than the sum of what each would be if the other were not present. Thus the principle of superposition should not be used with this type of flow.

Of course, the total quantities of runoff, plus storage and infiltration, must check the total volume of supply. The writer has made sure that the areas under each curve conform with this basic requirement. The function $\int q\, dT$ is readily determined mathematically for each of the curves in Fig. 27.

Curve 9, Fig. 28, is the rate-of-supply hydrograph that would yield the Hathaway runoff hydrograph, curve 2, Fig. 18, which is also shown in Figs. 27 and 28. Curve 10, Fig. 28, is the rate-of-supply hydrograph that would yield the Hathaway runoff hydrograph, curve 7, Figs. 27 and 28. These rate-of-supply hydrographs have been obtained by the differential equation:

$$\sigma = K\, y^2 + \frac{2}{3}\frac{dy}{dT} \qquad (8)$$

Values of $K\, y^2$ are the ordinates to curve 2 which, of course, was plotted by the Hathaway method—that is, by subtracting appropriate ordinates of curves like curve 1 from one another. Since values of $K\, y^2$ are known, corresponding values of y, and also values of $\frac{2}{3}\, y$, may be computed. These have been plotted as curve 11, Fig. 28. (Curve 12, Fig. 28, was plotted similarly, employing as a basis the ordinates of curve 7, Fig. 28.) Then the slope of curve 11 was determined graphically. These slopes constitute the values of the third term in the differential formula, Eq. 8. The last two terms in this equation were then added—with due regard to sign—and plotted as σ. The values of σ are the ordinates of curve 9, Fig. 28. (The ordinates of curve 10 were similarly obtained, using the slopes of curve 12.)

Such hydrographs of rate of supply—namely, curves 9 and 10—might just as likely occur as perfectly rectangular hydrographs, one of which is represented

in Fig. 27. Rains do not stop suddenly, but diminish slowly in intensity after their period of high rate, and sometimes are followed by a period of negative supply (or predominant percolation) somewhat as these curves might indicate. Thus the runoff hydrographs in Mr. Hathaway's paper appear correct for cer-

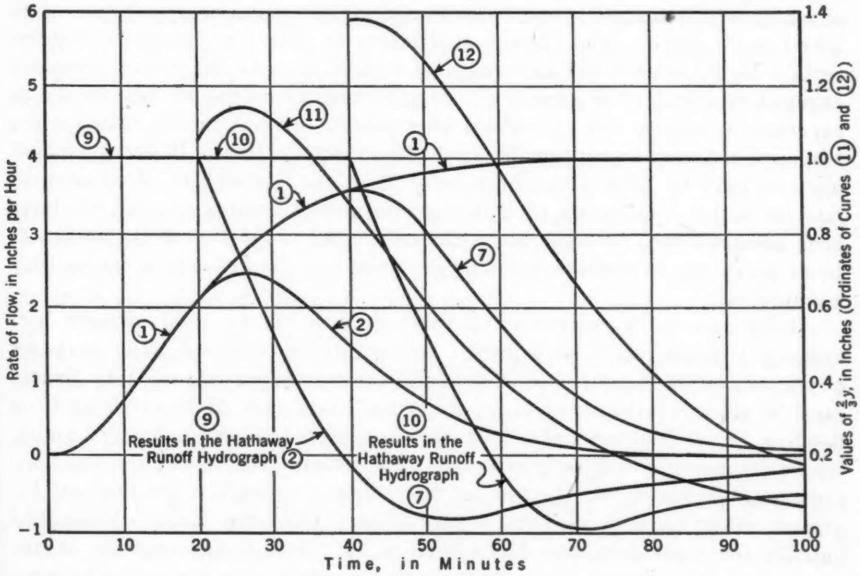


FIG. 28.—RATE-OF-SUPPLY HYDROGRAPHS (CURVES 9 AND 10) WHICH WOULD RESULT IN THE HATHAWAY RUNOFF HYDROGRAPHS (CURVES 2 AND 7)

tain plausible rate-of-supply hydrographs which might occur, although not for the simple rate-of-supply hydrographs which Mr. Hathaway seems to have assumed.

To obtain the runoff hydrograph for any arbitrary rate-of-supply hydrograph on the basis of the Horton differential formula, Eq. 8, two methods for analysis of the behavior of flood retention reservoirs are available: (1) The mass curve method presented by H. G. Payrow, M. Am. Soc. C. E. (26); and (2) the method (27) developed by the Miami Conservancy District, Dayton, Ohio.

DAVID S. JENKINS,[8] ASSOC. M. AM. SOC. C. E.[8a]—That part of the Symposium on airfield drainage, to which this discussion is largely devoted, is a major contribution from the science of hydrology to the advancement of both civil and military aviation. Mr. Hathaway provides the airport engineer with two important new tools for the design of storm drainage systems. First, by the use of Mr. Horton's (18) equations of overland flow as applied by

[8] Senior Airport Engr., Civ. Aeronautics Administration, Washington, D. C.

[8a] Received by the Secretary March 14, 1944.

W. W. Horner, M. Am. Soc. C. E. (27), for related purposes, an ingenious consolidation of Mr. Yarnell's (15) rainfall data into one very useful set of curves, and some bold assumptions concerning infiltration, a greatly improved method for calculating storm runoff from airfields is devised. Second, a simple and urgently needed procedure is presented by which the actual quantities of temporary pondage in areas between runways are computed from the established hydrograph. The correct application of these two methods requires certain highly specialized knowledge of hydrology. As the airport engineer engaged in a variety of general civil engineering duties cannot reasonably be expected to acquire this specialized information on short notice, delay in the adoption of these improvements should be expected. If the Hathaway paper were to have no other immediate value than the stimulation of widespread interest in the application of hydrology to airport drainage, much will have been accomplished. Furthermore, the occasional application of the tools will be of great and immediate value in the design of the airports to which they are applied.

Differences in requirements of drainage systems for civil airports and military airfields might be expected because of the different basic purposes of the two establishments. For obvious security reasons, the military airfield must be made serviceable on short notice and must offer minimum delay from flooding, but the commercial airport may be rendered unserviceable by flooding for longer periods with only economic loss. Counterbalancing the need for greater serviceability on the part of the military airfields is the demand for greater safety in civil airports arising largely from the factor of financial liability for passengers and cargo with which the military engineer is not confronted. Because of these compensating requirements, the writer believes that the five general criteria for design of the drainage system stated by Mr. Hathaway may be substantially adopted for civil airports. Postwar economic studies of aircraft movements, cargo values, and fuel reserves are needed to provide a more adequate basis for the design, not only of drainage structures but also of pavements and other airport facilities as well.

The method developed by Mr. Hathaway for calculating storm runoff from airfields is a definite improvement over the rational method, the Burkli-Ziegler equation, and other similar formulas which have little scientific foundation. That part of the rainfall which does not appear as runoff is quantitative and specific, and is not a ratio of the rainfall, as represented by a coefficient. As this truth has been well established, the subject is no longer controversial. However, the new method, like the old, requires a personal estimate of critical factors based on judgment and experience. Although separate evaluations of two or more unknowns will doubtless result in greater reliability than a collective estimate of the several, as embodied in the runoff coefficient, it is questionable whether the airport engineer is sufficiently familiar with the phenomenon of infiltration and with the roughness factors applicable to overland flow to make a correct estimate of them without additional guides. Mr. Horner (24*a*) states that possibly as many as 100,000 controlled experiments are available in the records of the Department of Agriculture from which infiltration capacities and overland flow characteristics are derivable. The prac-

tical application of these data to essential uses such as the design of military and civil airfields opens a new field to those familiar with these experiments. A contribution of practical value would be made, for example, if a table were prepared from these data showing the range in infiltration capacities for different soils and cover. The data could also be used in preparing a set of photographs showing grass covers representative of the ranges in the roughness coefficient n, similar to visual guides for channel roughness proposed by C. E. Ramser, M. Am. Soc. C. E. (28). If these suggested aids are not adopted, the data should be prepared from infiltrometer tests made on a representative group of airports.

In regions of high rainfall intensity where infiltration may constitute only a small fraction of the rainfall, large errors in estimating its value may result in only minor errors in design; but in regions of medium and low intensities this is not true. In the example given in the paper, Mr. Hathaway assumes an infiltration capacity of 0.6 in. per hr. This value is within the range of infiltration capacities—from 0.5 to 1.0 in. per hr—recommended in the *Engineering Manual* (a more detailed version of the Hathaway paper (23*a*)). From the writer's personal field observation of several thousand watershed rainstorms and the measurement of these and the runoff therefrom, together with the study of similar rainfall-runoff records and infiltrometer determinations, this range appears to be too narrow and its lower limits may be too high. Much greater emphasis could be placed on the influence of grass on infiltration. Unpublished infiltrometer records show wide variation in terminal infiltration capacities, but define a trend indicating that the infiltration capacity of a given soil having a good turf cover is roughly twice that of bare ground and that the over-all average of all terminal infiltration capacities on bare soil is roughly 0.2, with a range from 0.02 to 1.00 in. per hr or higher. Also unpublished are the results of numerous infiltrometer tests on high-plain prairie

TABLE 6.—TYPICAL TERMINAL INFILTRATION CAPACITIES, IN INCHES PER HOUR

Grass cover	Merita Clay Loam		Abilene Silty Clay Loam		Reagan Silty Clay Loam	
	Range	Mean	Range	Mean	Range	Mean
Poor.......	0.07 to 0.35	0.15	0.05 to 0.24	0.12	0.12 to 0.20	0.16
Good.......	0.20 to 0.70	0.38	0.23 to 1.64	0.73	0.22 to 2.65	1.26

soils of very low slopes, comparable to slopes of airports. Three of these are given in Table 6 to illustrate the broad range of infiltration capacities on some soils which may be expected by the airport engineer. The average time required to reach constant capacity in each case was as follows:

Soil	Time, in minutes
Merita clay loam..........	47
Abilene silty clay loam..........	76
Reagan silty clay loam..........	63

Since many airport engineers consider that the method proposed by Mr. Hathaway is not usable unless a guide for estimating infiltration capacities is provided, the tentative values in Table 7, for use in airport drainage design only, are suggested. The writer is fully aware that these values are subject to revision and that infiltration rates are influenced by many variables, but the use of Table 7 in conjunction with the Hathaway method should result in fewer and smaller errors than if no guide at all is followed. The values in Table 7 for dry antecedent conditions should be considered in regions where

TABLE 7.—ONE-HOUR INFILTRATION CAPACITIES, IN INCHES PER HOUR
(For Use Only in Design of Drainage Facilities for Airfields)

Character of entire soil[a] profile	DRY ANTECEDENT CONDITIONS		MOIST ANTECEDENT CONDITIONS	
	Dense grass	Poor grass[b] or bare	Dense grass	Poor grass[b] or bare
Tight plastic clays and silty clays.....	0.4 to 0.6	0.2 to 0.4	0.05 to 0.2	0.02 to 0.1
Porous loams and fine sandy soils.....	0.8 to 1.2	0.5 to 0.7	0.4 to 0.8	0.2 to 0.4

[a] Gravels and coarse sands not included. For pavement use zero. [b] To a small degree these values for bare soil will vary inversely with intensities. This is not true for dense turf.

high rainfall intensities occur only as summer thundershowers and where wet antecedent conditions will probably not prevail at the time of the design rainfall.

In calculating airport runoff to be used in designing the drainage system, an important source of error arises from the necessity of assuming in advance that a certain grass cover will ultimately be established on the area. No opportunity is afforded therefore to examine the grass cover on which the infiltration and roughness values—or the runoff coefficient—must be based. Airport turf is generally not well established until about the end of the first year, and in many cases not for two years following the completion of grading. During these two years which represent one design rainfall cycle, the land is relatively bare, runoff is high, the erosion of pavement shoulders advances rapidly, and the silting of drain lines may be serious. For this reason, the conservative results obtained by the Hathaway method seem further justified for the protection of the installation during this initial period, which may represent as much as 20% of the life of the pavements.

The practice of subtracting the infiltration capacity from the rainfall curve without a downward adjustment of the frequency is questionable, as it implies that the resulting supply curve retains the frequency of the original rainfall. Ground moisture conditions productive of the 1-hr infiltration capacity will not prevail at equal frequency and simultaneously with the design rainfall, although that is the interpretation generally placed upon the supply curve. A factor of safety is thereby introduced, the magnitude of which varies geographically. A similar interpretation is usually applied to other runoff formulas and probably accounts for the fact that some small and apparently inadequate airport drainage systems based on the 2-yr rainfall have not been overtaxed in eight or ten years of service. If the engineer desires to base

airport drainage design on the frequency of flooding (as must eventually become standard practice) and, if this design flooding frequency is two years, the selected design rainfall frequency will need to be increased, perhaps to one or even two storms per year, for all but the most important airports. Meanwhile, studies of the frequency of occurrence of selected infiltration rates and probabilities of groups and combinations of individual rainfall events should also be made from the wealth of available data previously mentioned. In any event, if an adjustment of the safety factor is necessary, it should be accomplished by revision of the selected frequency, and preferably not through modifications of other factors that might be misleading.

It is regrettable that the runoff methods proposed cannot be used for the many large intermediate airport areas having effective lengths of overland flow exceeding 600 ft, and there is some question whether the Horton equation would hold true on graded airport slopes 500 ft or 600 ft long. For applicability of that equation the surface (and adjacent surfaces) must be a true plane of such shape as to avoid channeling or accumulations of flow. Furthermore, as the relationship between length and critical time is very nonsensitive for the longer slopes, great refinement of results should not be expected in this range. For the larger areas a correct application of the rational method may be necessary, if indeed it is possible to apply that method correctly. Recent trial studies indicate that the Snyder Synthetic Unit Graph (29) may be adaptable to small watersheds of this type. Considerable success has also been achieved by numerous investigators in applying to small watersheds the "Modified Rational Method" as developed by Merrill Bernard, M. Am. Soc. C. E. (30), and others.

The method of determining the maximum volume of available pondage is not entirely clear although it is stated that this quantity depends upon land slope and permissible grades. General recommendations (31) for commercial airports of the three higher classes require that longitudinal and transverse grades should be limited to 1.5% maximum slope, and that the maximum change in grade at any point shall not exceed 2%. On the basis of these recommendations, in general, negligible pondage can be created along one drain line parallel to a runway when the longitudinal grade is greater than about 0.8% if the transverse grade is 1.5%. Fig. 26 affords favorable opportunity for the use of pondage as the longitudinal grades of the three runways are all less than 0.15%. For use as a guide in determining the maximum pondage requirements, the total runoff for each inlet can be obtained directly from Cols. 23 and 25 of Table 4(*b*).

Care should be taken to avoid the creation of ponding areas where the geologic structure is of such dip and character as to convey water from the pond to the pavement subgrade. A slight dip toward the runway with porous material outcropping in the pond and underlain by less pervious material presents a perfect opportunity for ponded water to reach the pavement base and subgrade. Borings made with this in mind will reveal such a condition if it exists.

Under "Computations of Storm Drain Capacities, Assuming Supplemental Ponding," Mr. Hathaway states:

"It is desirable to maintain the hydraulic grade line as near as practicable to the top of the drain pipe during the design storm."

This important statement is not further emphasized, and, although this requirement has apparently been satisfied in the example, the procedure by which it is accomplished in the case of ponds has not been clarified. If the hydraulic gradient is permitted to rise to the elevation of one of these ponds, discharge from the pond will cease. Backwater will occur from the drain line into the pond if the gradient is raised further. Fig. 26 shows rectangles beside each inlet which are not discussed, but other than these the only limitation placed on the discharge rate of each pond appears to be the flow capacity of the inlet grating.

To compute the runoff for storm drainage design (32), an adaptation of a rational method (33) is used by the Civil Aeronautics Administration which has successfully pioneered in the field of airport design:

$$Q = \frac{A\,I\,R}{T + t} \qquad (9)$$

in which Q = rate of runoff in cubic feet per second; A = area to be drained, in acres; I = percentage of imperviousness of the area; R = maximum average rate of rainfall over the entire drainage area, in inches per hour, which may occur during the time of concentration; T = duration of rainfall in hours (assumed to be 1 hour in airport design); and t = time allowed for removal of rainfall after the end of the storm (assumed as 2 hours in airport design). The value of R is the 1-hr, 2-yr rainfall intensity for all inlets.

As this method has been used in the design of many large airports, a brief comparison of the results obtained from it and from the Hathaway method is of interest. For this comparison two of the drain lines, C and D, of the midwestern airport in Fig. 29 have been investigated. These lines were selected because they are isolated, or branch, lines carrying neither water from other inlets nor "foreign" water from outside the airport. All computations for line D, including those of pondage, exclude any provision for future grading of the unfinished area to the west. The pondage along line D used in the application of the Hathaway method is the maximum that could be provided with the given inlet elevations. Because of the high longitudinal slope, no pondage can be created along line C. The high infiltration capacity of 0.9 in. per hr was selected to correspond as closely as possible for this rainfall region to the value 0.20 for "imperviousness" used in the method followed by the Civil Aeronautics Administration (33). The roughness factor represents average grass cover. The comparison shows the runoff computed by Mr. Hathaway's method to be appreciably greater, even with ponding, than that computed by Eq. 9. Since the soil surveys indicate that the area is quite sandy, the smaller quantities shown in Table 8 are apparently adequate. If the transverse grade along line D were to be increased to about 1%, pondage sufficient to contain the total design runoff for each inlet could be provided.

Neighboring property interests must be considered in connection with drainage. These interests, which may be seriously affected by the drainage

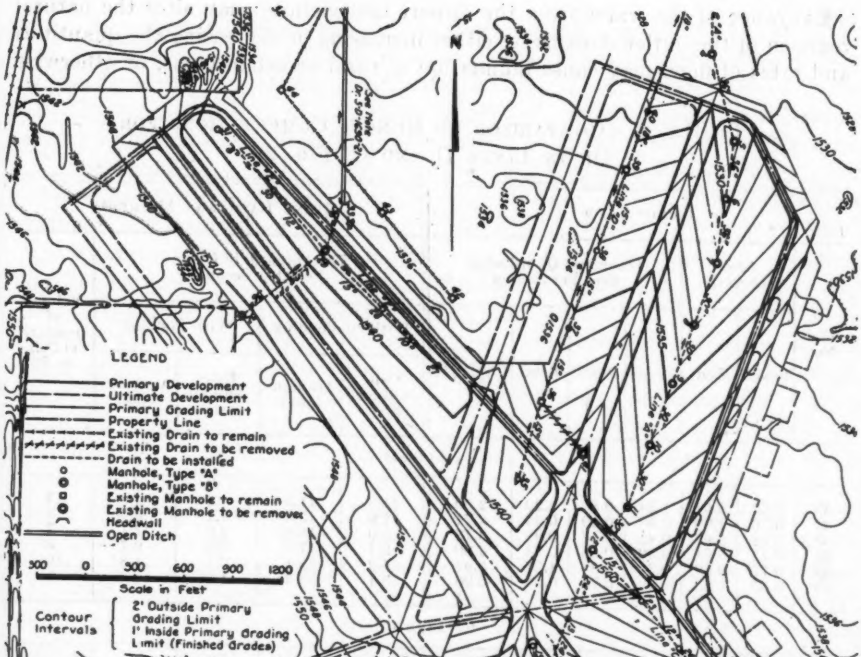

LEGEND
Primary Development
Ultimate Development
Primary Grading Limit
Property Line
Existing Drain to remain
Existing Drain to be removed
Drain to be installed
Manhole, Type "A"
Manhole, Type "B"
Existing Manhole to remain
Existing Manhole to be removed
Headwall
Open Ditch
300 0 300 600 900 1200
Scale in Feet
Contour Intervals
2' Outside Primary Grading Limit
1' Inside Primary Grading Limit (Finished Grades)

FIG. 29

system, may in turn affect the design. The possibility that the artificial conveyance of the water from the airport and vicinity may alter the natural regimen of the outlet streams by either increasing or decreasing the quantities and rates of flow, may cause inundation of rural or urban areas, or otherwise

TABLE 8.—COMPARISON OF RUNOFF COMPUTATIONS FOR DRAIN LINES D AND C, FIG. 29

Inlet No.	USING[a] EQ. 9						USING HATHAWAY METHOD[b]				
	Area (Acres)		Discharge Q, in Cubic Feet Per Second				Discharge Q, in Cubic Feet Per Second				Volume of pondage (1,000 cu ft)
							Without Ponding		With Ponding		
	Paved	Sod	Paved	Sod	Total	Accumulated	Individual area	Accumulated	Individual area	Accumulated	
(a) LINE D											
35	2.97	8.57	1.34	0.86	2.20	2.20	11.3	11.3	4.6	4.6	13
36	0.76	1.90	0.35	0.19	0.54	2.74	1.9	12.9	1.1	5.7	4
37	0.92	5.68	0.41	0.57	0.98	3.72	3.0	15.9	2.0	7.7	7
38	0.71	5.64	0.32	0.56	0.88	4.60	2.7	17.9	2.0	9.7	6
39	0.77	9.59	0.35	0.96	1.31	5.91	4.2	22.1	2.1	11.8	10
40	0.65	7.03	0.29	0.70	0.99	6.90	3.1	25.2	1.5	13.3	6
(b) LINE C											
26	0.63	1.94	0.28	0.19	0.47	0.47	1.9	1.9	....	....	....
25	0.63	3.02	0.28	0.30	0.58	1.05	2.0	3.8	....	....	....
24	1.16	7.22	0.52	0.72	1.24	2.29	4.4	8.0	....	....	....
23	1.72	7.61	0.77	0.76	1.53	3.82	6.1	13.7	....	....	....

[a] I equals 0.90 for pavements and 0.20 for turf; R = 1.50; T = 1; and t = 2. [b] The 1-hr infiltration capacity equals, for pavement, 0, and, for turf, 0.90; and the value of the roughness coefficient n equals, for pavement, 0.02, and, for turf, 0.40.

may damage private or public property should be investigated thoroughly before work is started. In his introductory remarks Mr. Hathaway indicates that the problem of diverting water from adjacent areas will be discussed. This water from neighboring lands can usually be disposed of most economically by interception in open channels located outside the area of operations. Then, the quantity of water reaching the final outlet channel under natural conditions should be calculated. As the results may ultimately be used for various purposes, including litigation, the runoff for the 5-yr, 25-yr, and 100-yr rainfall should also be computed and a permanent record made of all assumptions and estimates. The elevations of flood water in the final outlet channel under the natural condition should be computed by using the estimated quantities together with a simple application of the backwater method (34) introduced by the late H. R. Leach. Of interest in this connection is a recent case of an airport in the arid West where, because of neighboring property interests, it has been proposed to construct on the airport property a shallow reservoir of

large area with sufficient capacity to collect and evaporate the total annual runoff.

Where snow is cleared into a mound along the edge of the runway, the normal path of overland flow is blocked. The trapped water collects on the runway creating a dangerous operational hazard. Among the numerous means for disposing of this small discharge, a pre-cast concrete pipe of rectangular shape with removable lid and numerous surface inlets has proved satisfactory.

The California Bearing Ratio (CBR) is being used extensively in its original or modified form in the design of airport pavements. Nevertheless, there is a rather widespread misunderstanding of its purpose. As a result airport drainage is considered by some to be of little or no importance because, it is argued, flexible pavements will be designed on the basis of a saturated subgrade by the use of the CBR, and, therefore, flooding will cause no damage. This Symposium with its discussion of CBR, coupled with its emphasis upon airfield drainage, should aid in correcting this misunderstanding. For further clarification: First, the soaking procedure of the CBR does not necessarily produce saturation. Second, there is a vast difference between saturation and the surcharging of the subgrade through flooding. The construction of a stable pavement on a fluid subgrade would be virtually impossible regardless of the base course thickness, and the CBR does not anticipate this condition. Third, the principal cause of subgrade and pavement instability is the intermittent change in moisture conditions with the accompanying destructive alterations in volume and density. No better method than adequate drainage has yet been devised to minimize that change. Engineers may continue to build thicker and costlier pavements, but, regardless of thickness, strength, and cost, these pavements cannot be successfully floated on fluid subgrades or on those found where airport drainage is inadequate, which are swelling, shrinking, and frost active.

The drainage and pavement designs discussed in this Symposium as well as the plan shown in Fig. 29 are for the better classes of military and municipal airports. There is an urgent and rising demand for economy in airport construction, not only for the several thousand smaller cities but for the numerous smaller airports which will environ larger cities in the postwar era. To satisfy this demand the engineer will be compelled to seek less expensive runways and drainage facilities. The open channel will undoubtedly be used extensively for both surface and subsurface drainage, but an understanding of its limitations as well as of its usefulness is essential. The open channel may be widely adaptable if supplemented where necessary with small closed drains to remove those long depression flows that create dangerous, and frequently obscure, soft areas. Through the developments discussed in this Symposium and the current work of the Civil Aeronautics Administration and other agencies, engineers have or are obtaining much information needed for the design of pavements and drainage facilities to accommodate heavy craft at the large airports. They now must meet requirements for economy—on the smaller fields for the private flier—and to this end much investigational and development work remains to be done.

Bibliography.—

(15) **"Rainfall Intensity—Frequency Data,"** by David L. Yarnell, *Miscellaneous Publication No. 204*, U. S. D. A., 1935. (*a*) Fig. 29, p. 41.

(18) **"The Interpretation and Application of Runoff Plat Experiments with Reference to Soil Erosion Problems,"** by Robert E. Horton, *Proceedings*, Soil Science Soc. of America, Vol. 3, 1938, pp. 340–349.

(23) *Engineering Manual*, Constr. Div., Office of Chf. of Engrs., U. S. Army, 1943. (*a*) Chapter 21, Design of Drainage Facilities.

(24) **"Surface Runoff Determination from Rainfall Without Using Coefficients,"** by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), Fig. 5(*a*), p. 1052. (*a*) p. 1039.

(25) **"Storage and the Unit Hydrograph,"** by C. O. Clark, *Proceedings*, Am. Soc. C. E., November, 1943, Fig. 4, p. 1348.

(26) **"Sanitary Engineering,"** by H. G. Payrow, 1st Ed., International Textbook Co., Scranton, Pa., p. 84.

(27) **"Hydraulics of Steady Flow in Open Channels,"** by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941, p. 133.

(28) **"The Flow of Water in Drainage Channels,"** by C. E. Ramser, *Technical Bulletin No. 129*, U. S. D. A., November, 1929.

(29) **"Synthetic Unit-Graphs,"** by Franklin Snyder, *Transactions*, Am. Geophysical Union, 1938, p. 447.

(30) **"The Modified Rational Method of Estimating Flood Flows,"** by Merrill M. Bernard, *Low Dams*, U. S. National Resources Planning Board, 1938.

(31) **"Airport Design Information,"** *Bulletin*, Civil Aeronautics Administration, May 8, 1941, p. 13.

(32) **"Airport Drainage Design Information,"** *ibid.*, August 1, 1942.

(33) *Handbook of Culvert and Drainage Practices*, Armco Culvert Mfrs. Assn. 2d Ed., 1937, p. 187.

(34) **"Leach Backwater Method,"** *Handbook of Hydraulics*, by Horace W. King, 3d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 534.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A SYMMETRICALLY LOADED BASE SLAB ON AN ELASTIC FOUNDATION

Discussion

BY A. E. CUMMINGS, HARRIS EPSTEIN AND JAMES R. AYERS, JR., AND STANLEY U. BENSCOTER

A. E. CUMMINGS,[17] M. AM. SOC. C. E.[17a]—As a background for this subject the paper needs some reference to the extensive literature already available. Some of the works dealing with the problems of beams and plates on yielding foundations are indicated by the following references:

(1) **"Die Lehre von der Elasticität und Festigkeit,"** by E. Winkler, 1867, p. 182.

(2) Correspondence on **"Iron Permanent Way,"** by J. W. Schwedler, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, Vol. LXVII, 1882, p. 95.

(3) **"Über das Gleichgewicht Schwimmender Elastischer Platten,"** by H. Hertz, *Annalen der Physik und Chemie*, Vol. 22, 1884, p. 449.

(4) **"Über die Berechnung von Trockendocks,"** by O. Franzius, *Zeitschrift für Bauwesen*, 1908, p. 475.

(5) **Progress Report of the Special Committee to Report on Stresses in Railroad Track,** *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1191.

(6) **"Über das Gleichgewicht von Elastischen Platten unter einer Einzellast,"** by H. Happel, *Mathematische Zeitschrift*, Vol. 6, 1920, p. 203.

(7) **"Theorie des Trägers auf Elastischer Unterlage,"** by K. Hayashi, Julius Springer, Berlin, 1921.

(8) **"Belastingsproeven ter Bepaling van de Grondelasticiteit,"** by B. Wolterbeek, *De Ingenieur*, Vol. 36, 1921, p. 501.

(9) **"Vorlesungen über Technische Mechanik,"** by A. Föppl, 4th Ed., Vol. V, Leipzig, 1922.

(10) **"Über den Balken auf Nachgiebiger Unterlage,"** by K. Wieghardt, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 2, 1922, p. 165.

(11) **"Platten Rechteckiger Grundriszteilung auf Elastisch Nachgiebiger Unterlage,"** by Lewe, *Der Bauingenieur*, 1923, p. 453.

NOTE.—This paper by Stanley U. Benscoter, Jun. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. Robert O. Jameson, and L. J. Mensch; November, 1943, by Alfred L. Parmé; and February, 1944, by A. A. Eremin.

[17] Research Engr., Raymond Concrete Pile Co., New York, N. Y.

[17a] Received by the Secretary February 18, 1944.

(12) **"Om Beregning af Plader paa Elastisk Underlag,"** by H. Westergaard, *Ingeniøren*, Vol. 32, 1923, p. 513.
(13) **"Kreisplatten auf Elastischer Unterlage,"** by F. Schleicher, Julius Springer, Berlin, 1926.
(14) **"Strength of Materials,"** by S. Timoshenko, Pt. II, Van Nostrand, 1930, p. 401.
(15) **"Bodenpressung und Bettungsziffer,"** by K. Terzaghi, *Österreichischen Bauzeitung*, June 18, 1932.
(16) **"Die Bemessung von Flachgründungen aus Eisenbeton und die Neuere Baugrundforschung,"** by O. Froelich, *Beton und Eisen*, No. 12, 1935.
(17) **"Stresses and Deflections in Loaded Rectangular Plates on Elastic Foundations,"** by G. Murphy, *Bulletin No. 135*, Iowa Eng. Experiment Station, Ames, Iowa, 1937.
(18) **"Bending of an Infinite Beam on an Elastic Foundation,"** by M. Biot, *Journal of Applied Mechanics*, March, 1937.
(19) **"Über den Träger auf Elastischer Unterlage,"** by K. Marguerre, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 17, 1937, p. 224.
(20) **"Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth,"** by A. Hogg, *Philosophical Magazine*, London, Vol. 25, 1938, p. 576.
(21) **"Thin Plates on Elastic Foundations,"** by D. Holl, *Proceedings*, Fifth International Cong. for Applied Mechanics, Cambridge, Mass., 1938, p. 71.
(22) **"Mathematical Methods in Engineering,"** by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940, p. 304.
(23) **"Theory of Plates and Shells,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., 1940, p. 248.
(24) **"Mathematical Methods in Engineering,"** by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940, p. 35.
(25) **"An Introduction to the Theory of Elasticity for Engineers and Physicists,"** by R. Southwell, Oxford Univ. Press., 2d Ed., 1941, p. 215.

The author's approach to the problem under discussion appears to be more complex than is necessary. In order to find a solution, the author makes use of slope deflections, moment distribution, and elastic theory based on the differential equation of the elastic curve of the deflected slab. Actually, the problem can be solved by the elastic theory alone with considerably less mathematical work than is involved in the author's solution.

The load condition represented by Fig. 7(*a*) was solved years ago. It is a problem involving a beam resting on a yielding foundation and loaded with two concentrated loads symmetrically placed with respect to the center line. This is just a special case of the problem of the railroad tie which was the principal subject of the early investigations of E. Winkler (Reference 1), J. W. Schwedler (Reference 2) and others. The only difference between Fig. 7(*a*) and the railroad tie is the fact that the concentrated loads on the tie are some

distance in from the ends of the tie, whereas the loads in Fig. 7(*a*) are at the ends of the beam. However, the problem with the loads at the ends of the beam has been solved and the complete solution was given by K. Hayashi (Reference 7).

Fig. 7(*b*) refers to the more complicated condition when the backfill is in place and there are various forces and moments acting on the base slab. In order to solve this problem as represented by the force diagram of Fig. 2, the author has introduced slope-deflection equations, beam functions, fixed-end functions, and end-rotation functions. None of these is needed for the solution of this problem, as it is possible to solve the entire problem at one stroke by means of the differential equation of the elastic curve of the beam.

The general solution of Eq. 13 is Eq. 17 which contains four arbitrary constants of integration. The author has chosen to set two of these constants equal to zero for reasons of symmetry and he has then determined the other two constants by the boundary conditions in Eqs. 19. It is this procedure that necessitates the introduction of all those other functions into the analysis. Actually, it is possible to solve the entire problem by means of Eq. 17. To do this, it is only necessary to find the four boundary conditions that will determine the four constants A, B, C, and D. For this purpose the following four boundary conditions are available:

(1) By symmetry, the elastic curve of the deflected slab has a horizontal tangent at the center line. Therefore:

$$\left[\frac{dz}{dx}\right]_{x=0} = 0 \qquad \text{(49a)}$$

(2) The bending moment at the right-hand end of the slab is $-M_a$. Therefore:

$$\left[\frac{d^2z}{dx^2}\right]_{x=a} = -\frac{M_a}{E\,I} \qquad \text{(49b)}$$

(3) The shear at the right-hand end of the slab is $-F$. Therefore:

$$\left[\frac{d^3z}{dx^3}\right]_{x=a} = -\frac{F}{E\,I} \qquad \text{(49c)}$$

(4) The summation of vertical forces must be zero. Therefore:

$$-F = \int_0^{0.5L} w\,dx = -k\int^{0.5L} z\,dx \qquad \text{(49d)}$$

When the four integration constants of Eq. 17 have been determined by means of these four boundary conditions, the resulting equation will be a complete solution of the problem. This equation will contain all of the external forces shown in Fig. 2, and it will contain the elastic coefficients of the soil and the slab. The amount of mathematics required to present the solution would be about the same as that involved in the author's Eqs. 12 to 22, inclusive. The slope-deflection equations, beam functions, fixed-end functions, and

end-rotation functions are all unnecessary. The problem of superimposing that external bending moment on the general solution of Eq. 13 is quite simple. Hayashi (Reference 7) devotes only a short paragraph to the subject and merely remarks that the external bending moment can be included in the solution without difficulty.

The author has not offered any explanation as to why he solved this problem on the basis of the incomplete force diagram shown in Fig. 2. A structure of the type under discussion, with the channel empty and with the backfill in place outside the walls, is shown in Fig. 16(*a*). The horizontal thrust, *P*, is

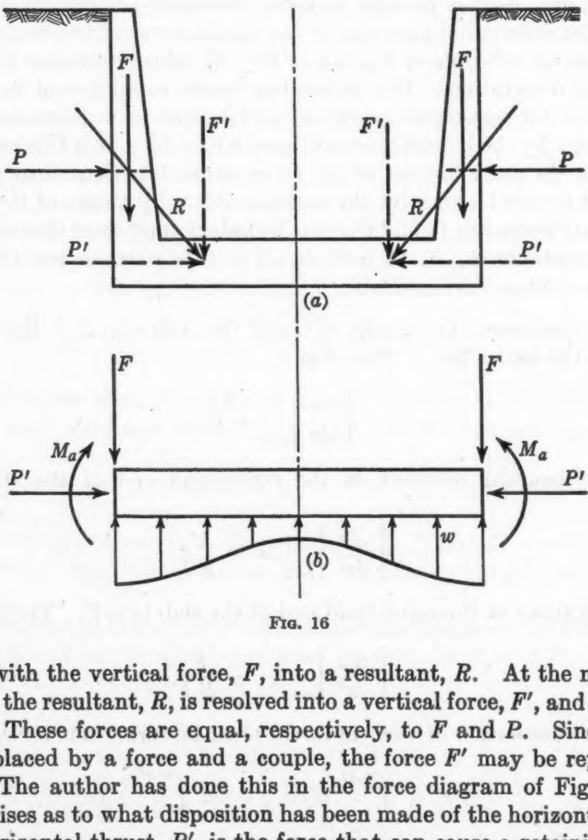


FIG. 16

combined with the vertical force, *F*, into a resultant, *R*. At the middle plane of the slab, the resultant, *R*, is resolved into a vertical force, F', and a horizontal force, P'. These forces are equal, respectively, to *F* and *P*. Since any force may be replaced by a force and a couple, the force F' may be replaced by *F* and M_a. The author has done this in the force diagram of Fig. 2, but the question arises as to what disposition has been made of the horizontal force, P'.

This horizontal thrust, P', is the force that can cause a retaining wall or a dam to fail by sliding even though the structure may be safe against overturning due to the eccentric force, F'. In the design of the side-wall itself, this force, P', is taken into account because it produces a shearing stress on the horizontal plane where the base of the wall rests on the slab. The design of the bottom of the wall includes an investigation of three kinds of stresses: (*a*) The direct compression produced by *F*, (*b*) the flexural stresses produced by M_a, and (*c*) the shear produced by P'. Accordingly, when the structure as a whole is being

considered, the diagram of external forces should include two equal and opposite horizontal thrusts, P', applied to the edges of the slab as shown in Fig. 16(*b*).

The probable importance of the forces, P', would depend on the dimensions of the structure. In a shallow structure with a thick base slab, the effect of P' would be negligible in comparison with the effects of F and M_a. In a deep structure with a thin base slab, the force, P', might materially increase the bending moments in the slab. In all cases, these forces will exist in any actual structure of this type and they should not be discarded unless it can be demonstrated that their effect is negligible. Furthermore, it should be kept in mind that the slab is bent by transverse loads and bending moments so that these horizontal thrusts are applied to the edges of an already deflected slab.

Eq. 13 is not satisfactory for this problem because it does not include the effect of the horizontal forces, P'. In order to include these forces, it is necessary to start with the differential equation:

$$\frac{d^4z}{dx^4} + \frac{P'}{E\,I}\frac{d^2z}{dx^2} + \frac{k}{E\,I}z = 0 \qquad (50)$$

This is a linear differential equation of the fourth order with constant coefficients. Methods of solving the equation are well known (24) and the general solution may be written in the form

$$z = \sum_{n=1}^{n=4} A_n\, e^{xp_n} \qquad (51)$$

in which A_n represents the four arbitrary constants of integration, p_n represents the four roots in the characteristic equation, and e is the Napierian log base. The four values of p_n are obtained from the equation

$$p^4 + \frac{P'}{E\,I}p^2 + \frac{k}{E\,I} = 0 \qquad (52)$$

After these values of p_n are substituted into Eq. 51, the four values of A_n are determined by means of the boundary conditions given in Eqs. 49. The solution of Eq. 50 is somewhat more laborious than that of Eq. 13, but the use of Eq. 50 with the proper boundary conditions will yield a complete solution without the aid of slope deflections or moment distribution or any other form of structural analysis.

When a mathematical analysis of this kind is applied to an engineering problem it is desirable to keep in mind the fact that the mathematics is only a part of the solution of the problem. All problems of this type involve important practical considerations which are easily overlooked in the maze of mathematics with which the problem is surrounded. Any one contemplating the use of some of these equations as design formulas should understand that this entire analysis is based on certain definite assumptions which may or may not be fulfilled in the actual structure. For example, in connection with Eq. 1, the author has defined the foundation modulus, k, as the constant of proportionality between the upward reaction, w, and the downward displacement, z. Nowhere in the paper is there any mention of the fact that k is supposed to

work both ways. In other words, if the bending stresses in the slab tend to lift the slab from the ground, the mathematics require a tensile stress to be generated between the soil and the underside of the slab. This tensile stress is a downward reaction which is proportional to the upward displacement of the slab and the constant of proportionality is the same k that is in Eq. 1.

This idea can be expressed mathematically by the following statement: There is no restriction on the sign of z in Eq. 17. Positive values of z refer to downward displacements, and negative values of z refer to upward displacements. The stress between the soil and the slab may be either tension or compression in direct proportion to the displacement and the constant of proportionality is the foundation modulus, k, in either case. The mathematical analysis therefore requires that k must be able to act in tension or compression.

In contrast with this mathematical requirement, practical considerations lead to the conclusion that the soil cannot exert tension on the underside of the slab in the prescribed manner. If the slab were supported by a pile foundation, the piles would provide anchorage against uplift as well as resistance to downward displacement. However, even under such conditions the elastic behavior of piles subjected to uplift forces would not necessarily be the same as their elastic behavior under downward forces. The analysis of a slab supported by piles which could also act as anchors would require two different values of k. When the slab simply rests on earth or rock, there is no restraint to hold it down if the external forces try to bend it up off the ground.

The author has remarked (see heading, "Design Formulas") that "a practicing engineer instinctively prefers to deal entirely with bending moment values in a structure analysis." In the problem under discussion, it is not safe to trust entirely to instinct and to stop the investigation after the bending moments are determined. The displacements must be investigated also because the entire mathematical analysis is useless if there is any part of the slab for which the value of z is negative. This is a matter of considerable practical importance and it has been discussed thoroughly by other writers, particularly by Hayashi (Reference 7). The author's analysis does not include the final equation for the elastic curve of the deflected slab. However, this equation should be derived and used in the investigation of the structure.

Another practical aspect of this problem is the numerical value of k. The author has suggested load tests to determine the value of k but he recommends a value of 100 lb per cu in. for design purposes in cases where load test results are not available. The writer finds it difficult to accept such a recommendation for several reasons. In the first place, it is quite a problem to get a suitable value for k even when careful load tests are made. The literature in the foregoing list contains numerous demonstrations of this fact. A series of load tests with different sizes of bearing areas on the same soil would give a different value of k for each bearing area. The choice of k for the full size structure would therefore be a matter of judgment at best. In the second place, the numerical values of k that are obtained from load tests on different types of soil will vary over a considerable range. There appears to be no justification for the assumption that 100 lb per cu in. would be a suitable value for all soils merely because no load tests were made. Furthermore, an arbitrary assump-

tion of this kind is hardly compatible with the powerful mathematical analysis the author has applied to the problem.

In conclusion, it seems desirable to make the following observations: Problems of this kind have engaged the attention of engineers for many years and complete mathematical solutions are already available for many of them. In general, these mathematical solutions are much more concise and elegant than the cumbersome procedure used by the author. The real need in connection with these problems is a better understanding of the physical significance of the equations and a considerable number of field measurements on actual structures to find out how accurately the mathematics can predict the behavior of the structure. As the matter stands now, the abstract mathematics has run far ahead of the practical side of the problem.

HARRIS EPSTEIN,[18] ESQ., AND JAMES R. AYERS, JR.,[19] ASSOC. M. AM. SOC. C. E.[19a]—The assumption of the proportionality between distributed soil reactions and the deflection of the distributing medium has been used by the writers since 1920.

A particular application of this assumption was made recently, at the suggestion of Ben Moreell, Hon. M. Am. Soc. C. E., to the design of graving docks which consist of two concrete side-walls connected by a floor slab resting either directly on the underlying soil, or on bearing piles, depending on the foundation conditions at a particular site. The side-walls are usually backfilled with earth. A typical cross section appears in Fig. 17, which is quite similar to that of the water channel in Fig. 6.

The side-walls of the water channel, as well as those of a dry dock, are of such height that, in comparison with the floor thickness, they possess almost infinite stiffness in the vertical direction and cannot, therefore, have a curved elastic line. This fact leads to the logical conclusion that the parts of both the force and displacement diagrams, directly under the walls, should be straight lines tangent to the curved part of these diagrams for the floor proper at the intersection of the floor with inner faces of side-walls and extending to the outer faces, as shown in Fig. 17. This contrasts with Fig. 2 in which the curved lines of the force and displacement diagrams extend under the side-walls and terminate at their centers. The omission of the forces on the outer halves of the wall width is certain to influence the magnitude of the moments in the floor slab, the influence increasing with increase in the width of the walls. Therefore the force and displacement diagrams should extend across the entire width of the structure.

Let: W_s = weight of the part of the side-wall above the top of the floor slab; and a' = distance from the inner face of the side-wall to the center of gravity of the wall. The effect of the downward distributed load of the side-wall may be incorporated in the solution of the author's fundamental differential equation by applying to the slab, at the inner face of the side-wall, a concentrated load, W_s, and a moment of magnitude $W_s\, a'$. Since the reaction diagram

[18] Prin. Designing Engr., Bureau of Yards and Docks, U. S. Navy Dept., Washington, D. C.

[19] Senior Designing Engr., Bureau of Yards and Docks, U. S. Navy Dept., Washington, D. C.

[19a] Received by the Secretary March 13, 1944.

under the wall is a straight line, the shear and moment, as well as the slope of the elastic curve, at the inner face of the side-wall may be expressed in terms of the foundation reactions at the inner and outer faces of the wall. These three expressions, combined with the conditions of symmetry and the expression for foundation reaction at the inner face of the side-wall, are sufficient for

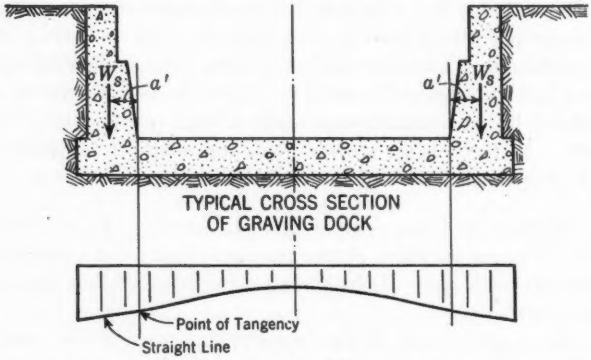


FIG. 17.—FORCE AND DISPLACEMENT DIAGRAM

evaluating the constants for the general solution of the fundamental differential equation as applied to structures of this shape. Then the expressions for moment, shear, and foundation reaction at any point may be written and used to compute values of these quantities directly, thus eliminating the necessity for balancing the moments by the indirect method suggested by the author.

In the illustrated example of the water channel, the writers check the author's value of 48 kip-ft for the moment at the center of the slab, assuming a side-wall width of zero; whereas, if the force diagram extends across the entire wall width as a straight line, the moment at the center of the slab reduces to 40 kip-ft, the difference being about 16%.

The maximum value of the negative moment (tension in the top of floor slab) does not always occur at the center of the slab width. In many cases, for large values of α, there are two points of zero shear in the slab in addition to the one occurring at the center. The negative moment, which extends across the entire slab between side-walls, then has a minimum value at the center of the slab, and equal maximum values on either side of the center at the other two points of zero shear. This is due to the fact that the computed distributed reaction acts downward for a part of the slab adjacent to the center of the structure, thus requiring a force to prevent lifting of the slab from the foundation. The dead weight of the slab is usually more than enough to supply this force in the case of a dry dock.

STANLEY U. BENSCOTER,[20] JUN. AM. SOC. C. E.[20a]—The comments and information in the discussions are appreciated by the writer and will render useful service to the future reader as viewpoints gained from varied experience. The

[20] Associate Engr., Bureau of Aeronautics, Navy Bldg., Washington, D. C.

[20a] Received by the Secretary February 4, 1944.

proper assumption as to reaction distribution in the design of a flexible base slab has always been a controversial issue. The writer has seen several types of broken-line diagrams which give a minimum value of base reaction at the center and a maximum value at the edge. Mr. Mensch has illustrated the use of a parabola over part of the half span; and Mr. Eremin, the use of a sine curve. None of these approximate methods of treating the problem contains any rational association with a physical measure of the foundation rigidity. Likewise, none of these approximate methods gives any suggestion as to the increased stiffness of the base slab, which is caused by the foundation and which must be used in an elastic indeterminate structural analysis.

Since the foundation modulus is a readily obtainable measure of the rigidity of the foundation, a method of analysis should be employed which gives results that are directly associated with this modulus. Thus, a rational analysis of the structure is obtained.

Various methods of analysis should be recorded so that calculated results can be compared with experimental field measurements whenever such measurements become available. A stress analyst can only express a plea that future plans for reinforced concrete structures include the installation of strain gages and pressure cells for measuring both reactions and acting earth or water pressures.

Inconsistently, Mr. Mensch objects to using the foundation modulus in a rational analysis of the slab, yet proceeds to use it in calculating the angle of rotation of the side-wall in Fig. 13. The writer wishes to recommend that structural engineers acquire the habit of calculating the angle of rotation of a retaining wall as presented by Mr. Mensch. The acting pressures are closely related to this angle. Although Mr. Mensch neglects the shear through the base slab at the section of the water stop, this shear affects the base reaction and the angle of rotation of the wall. Inclusion of this shearing force permits a reduction in the length of the heel. The writer has never found the loading condition mentioned by Mr. Mensch critical. Stresses are usually at a maximum after the backfill has been placed and before water has been allowed to enter the channel. The critical condition for stability occurs with the channel partly full and must be determined by trial and error. The writer does not extend the base slab beyond the side-wall as illustrated by Mr. Mensch in Fig. 12 when the wall and slab are constructed as a monolithic frame.

The graphs of moment factors presented by Mr. Jameson are a valuable addition to the paper. The writer is inclined to believe that the majority of designers will prefer to calculate internal moments as suggested by Mr. Jameson rather than as presented in the paper. Calculating the simple beam moments and correcting them for the effect of total end moment will resemble more closely the usual analysis of beams with fixed loading. The moment factors for a simple beam would be more instructive if they were plotted as coefficients of the moment $F\,a$ rather than as coefficients of $F\,\lambda$. Thus, the coefficients of $F\,a$ would reflect completely the effect of the foundation rigidity, and the curve for $\alpha = 0$ would be the well-known parabola for uniform loading. A complete set of design information would consist of three families of graphs for z, V, and

M for a simple beam and a similar set of three graphs for the effect of a unit end moment.

Mr. Parmé has recalled the close relationship between the floating slab theory of an elastic slab on an elastic foundation and an element of an elastic cylinder. The use of moment distribution in cylindrical tank analysis is illustrated in the reference[13] given by Mr. Parmé. The writer has used moment distribution for the analysis of cylindrical tanks and elastic base slabs for several years. In cylindrical tanks, bottom moments and shears generally produce little effect at the top of the cylinder. However, many thick-walled water tanks, built in army camps, designed to conserve steel, showed considerable effect at the top due to bottom moments. It is rather uncommon to find a base slab in which the edge moments and shears produce only a negligible effect at the center.

Mr. Eremin's analysis shows the error of neglecting the effect of foundation rigidity on the stiffness of the base slab. The sine curve gives a reasonable shape for the reaction distribution. The minimum ordinate of the diagram should be associated in some rational manner with the foundation rigidity.

The purpose of the paper is to demonstrate that an indeterminate structure resting on an elastic foundation can be analyzed by moment distribution. The mathematical development was planned and completed with this objective in view. An illustration of the analysis of an indeterminate structure was given in Fig. 8. As a matter of secondary importance, since the solution is well known (as explained by Mr. Cummings), it was shown in Fig. 7 that a determinate structure can also be analyzed by using the same graphs and formulas that are required for the indeterminate case. This inclusion of a determinate case appears to have sidetracked the attention of Mr. Cummings from the primary accomplishment of the paper. The methods of analysis which he presents as being preferable deal only with the determinate case.

Mr. Cummings opens and closes his discussion with statements that "The author's approach to the problem under discussion appears to be more complex than is necessary" and "these mathematical solutions [existing solutions] are much more concise and elegant than the cumbersome procedure used by the author." His entire discussion, which lies between these two criticisms, deals only with a determinate structure and offers the designer no suggestion as to how an indeterminate structure might be analyzed. This fact is particularly significant for the reader who may wish to evaluate the validity and applicability of the criticisms. A more concise method of analyzing the indeterminate case is not to be found in so far as the writer is aware.

Mr. Cummings explains that the complicated functions of the paper arise because the author "has chosen to set" two of the integration constants, C and D, equal to zero. Actually, the solution is unique and the writer had no personal choice in the matter. Eqs. 49, offered by Mr. Cummings in defense of his claim actually disprove it because they show immediately that C and D must be zero. To prove this statement, it is desirable to simplify Eq. 49*d*.

[13] "Circular Concrete Tanks Without Prestressing," *Concrete Information Sheet No. ST 57*, Portland Cement Assn.

With the assistance of Eq. 49*c*, Eq. 49*d* may be reduced to:

$$\left[\frac{d^3z}{dx^3}\right]_{x=0} = 0 \qquad (53)$$

Eq. 53 is recognizable as a condition of symmetry which should be used in place of Eq. 49*d* since it involves considerably more mathematical labor to integrate than to differentiate a sum of products of transcendental functions. Also the third derivative must be computed for use in Eq. 49*c* and thus is available for use in Eq. 49*e*. Substitution of the general formula for z, as given by Eq. 17, into Eqs. 49*a* and 53, which are equations of symmetry, or into Eqs. 49*a* and 49*d* gives,

$$C + D = 0 \qquad (54a)$$

and

$$C - D = 0 \qquad (54b)$$

Eqs. 54 show that C and D are zero. The remainder of the solution for the determinate case consists of solving Eqs. 49*b* and 49*c* simultaneously for A and B, corresponding to the procedure used by the writer for solving the indeterminate case. For that particular determinate case in which V_a and M_a are known, Eqs. 49*b* and 49*c* are correct. For the indeterminate case which should be solved by moment distribution, Eqs. 19*a* and 19*b* give a correct statement of the boundary conditions. There are six possible ways of stating the boundary problem, since there are six combinations of four things taken two at a time. In all six cases C and D must be zero because of symmetry.

The effect of thrust on bending was purposely omitted to conform to standard practice in the design of rigid frames of reinforced concrete. Since the eccentricity of the thrust is several hundred times the maximum relative deflections, the effect of thrust on bending is commonly regarded as negligible in the design of concrete frames. This matter becomes of considerable importance in thin, flexible structures such as those used in the monocoque construction of airplanes.

Mr. Cummings expresses concern over the tensile reactions which may occur at the center of the slab. He states that "the entire mathematical analysis is useless if there is any part of the slab for which the value of z is negative." The writer considers this to be a serious exaggeration of a trivial matter. Tensile reactions occur with flexible beams on rigid foundations, that is, with $\alpha > 2$. Such a case is a concrete conduit on a limestone foundation when the rock has a modulus of elasticity of several million pounds per square inch. The negative calculated reactions must exceed the weight of the slab before a theoretical error occurs. Actually, on such rigid foundations, floating slab theory can only give an approximate value for maximum moments. The error is not due to the slight tensile reactions at the center of the span but, rather, to the need for a more accurate influence relationship between deflections and reactions than that given by floating slab theory.

This paper should be regarded as an introduction to the use of floating slab theory in indeterminate structures. Extension of the concepts to more complicated cases remains for future study.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ILLINOIS WATER LITIGATION, 1940-1941

Discussion

By H. P. Ramey, and Langdon Pearse

H. P. Ramey,[7] M. Am. Soc. C. E.[7a]—In Table 7, Mr. Pearse indicates that remedial measure No. 9 was the initial suggestion of the State of Illinois. There was some agreement as to the desirability of budgeting the diversion, but the initial suggestion for it did come from the experts for the State of Illinois. Budgeting of the diversion was started long before the 1940 hearings. As assistant chief engineer of The Sanitary District of Chicago, the writer testified on September 9, 1940, that the flow began to be budgeted very carefully in January, 1939, when the diversion was reduced to 1,500 cu ft per sec. At the same time a tabulation was introduced as Illinois exhibit No. 40, which showed the budgeting, by months, from January, 1939, through 1940, and actual results through July, 1940.

The representatives of the lake states presented their first testimony regarding remedial measures on September 12, 1940.

Langdon Pearse,[8] M. Am. Soc. C. E.[8a]—Following the denial of the petition, work on the additional final settling tanks at the West-Southwest works progressed to about 85% completion, before coming to a stop. Of the three turbo-blowers under contract on August 29, 1940, only one unit was delivered late in 1941 and erected in 1942. However, it was not operated until April, 1943, because of the impossibility of securing necessary gate valves. Twelve contracts were let in 1940–1941 on the construction program to enlarge the Southwest works and provide facilities for treating the effluent of the West Side works. This group of contracts as a whole was only about 22% completed in the fall of 1941, when World War II began. At the end of 1943, construction practically ceased. On some contracts, nothing had been done. As yet (1944) there is no sign of relaxation in priorities, allowing work to be resumed. The

Note.—This paper by Langdon Pearse was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1943, by Willem Rudolfs, and June, 1943, by L. R. Howson, A. M. Buswell, and Lloyd M. Johnson.

[7] Asst. Chf. Engr., The San. Dist. of Chicago, Chicago, Ill.

[7a] Received by the Secretary February 16, 1944.

[8] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

[8a] Received by the Secretary February 26, 1944.

completion of the necessary permanent facilities to dispose of the entire production of sludge appears to be postponed indefinitely, until peace comes.

To provide temporarily for the handling of the excess sludge from the West-Southwest plants, plans were drawn for a 16-in. force main and seventy-eight acres of lagoons, about 16 ft deep, divided into sixteen units. The lagoons are located about 4.75 miles down channel from the Southwest works, between the Des Plaines River and the Main Channel. On January 22, 1942, a contract was let for 1,300 tons of cast-iron pipe. This was delivered in May, 1943. Other contracts were let in 1942 for the valves, sludge pumps, and lagoon construction; and, in 1943, for the pipe laying. The pipe line and lagoons were ready for service on November 1, 1943.

Until additional equipment and the requisite structures can be built to complete the Southwest works and supplement the West Side works, there is little chance of meeting the desired goal of a high-grade effluent from all the sewage treatment works of the Sanitary District.

The load reaching the four major treatment works of the Sanitary District is shown in Cols. 1 and 4, Table 11, based on population equivalents derived

TABLE 11.—PERCENTAGE REMOVAL FROM FOUR MAJOR SEWAGE TREATMENT WORKS

Period	YEARLY AVERAGE, PERCENTAGE REMOVAL			AVERAGE PERCENTAGE REMOVAL IN JUNE, JULY, AND AUGUST		
	Population equivalent (1)	5-day B.O.D. (2)	Suspended solids (3)	Population equivalent (4)	5-day B.O.D. (5)	Suspended solids (6)
1939	....	78.2[a]	82.0[a]	....	76.9[c]	82.0[c]
1940	6,200,000[b]	67.8	72.5	5,735,000	66.6	72.7
1941	6,512,000	60.6	67.5	6,540,000	61.3	69.9
1942	6,693,000	61.5	65.7	6,336,000	65.7	69.5
1943	6,594,000	57.5	62.1	6,417,900	56.0	60.1

[a] Includes Southwest works, July to December, inclusive. [b] April to December, inclusive. [c] Includes Southwest works, July and August only.

from the B.O.D. actually reaching the works. The 1940 census population of the Sanitary District was 3,962,514. From a total of 4,581,111 in 1940 (Cook County representing 4,063,342) the U. S. Census Bureau estimates a decrease of 32,113 (approximately 0.7%) in the population of Cook, Du Page, and Lake counties in Illinois and Lake County, Indiana, between April 1, 1940, and March, 1943.

The results of this population decrease on the Southwest works (Table 3) and the effect on the over-all efficiency of the four major works are explained in Table 12 which shows the apportionment of incoming flow to sewage receiving preliminary settling only and to sewage receiving complete treatment. Under present working conditions, the sewage receiving activated sludge treatment at the Southwest works is limited to that producing the required sludge for dewatering and drying for fertilizer, when added to the North Side sludge.

The over-all efficiency of the four major sewage treatment works of the Sanitary District is illustrated by the data in Cols. 2 and 3, Table 11. The averages for the three months of June, July, and August in Cols. 5 and 6, Table 11, are but slightly different.

TABLE 12.—APPORTIONMENT OF FLOW (IN MILLION GAL PER 24 HR), SOUTHWEST WORKS

Year or period	Prelimi- nary only	Activated sludge	Total
1939[a]	0	282	282
1940	121	192	313
1941	184	154	338
1942	188	143	331
1943	209	143	352

[a] July to December, inclusive.

The condition of the flow from the Main Channel as it enters the Brandon Pool is reflected by the analyses at Lockport. The yearly averages are shown in Table 13(*a*) and the averages for June, July, and August are given in Table 13(*b*). Samples are not collected at the outlet end of the pool.

From September 15, 1940, to November, 1942, no sludge was discharged from the treatment works into the channels of the Sanitary District or into the tributary rivers. Because of the 10-day test in 1940, a large amount of previously deposited sludge was swept down into the Brandon Pool. Apparently conditions in the pool were somewhat worse in the summer of 1941 than in 1940; 1940 was better than 1939 or 1941. In 1942, the condition was somewhat improved over that in 1941, except that an early spring in 1942 caused nuisance conditions at Lockport in April and May, whereas in 1941, the worst month was June. In 1942, from April through November, there was no oxygen (or only a trace) present in the flow at Lockport. In 1943, the dissolved oxygen disappeared in May, and reappeared in October.

TABLE 13.—DIVERSION FROM LAKE MICHIGAN AND DISCHARGE AND ANALYSES AT LOCKPORT, ILL., 1936 TO 1943, INCLUSIVE

Year	(*a*) YEARLY AVERAGE				(*b*) AVERAGE, JUNE, JULY, AND AUGUST			
	Cubic Feet Per Second		Parts Per Million		Cubic Feet Per Second		Parts Per Million	
	Diversion	Discharge	Dissolved oxygen	5-day B.O.D.	Diversion	Discharge	Dissolved oxygen	5-day B.O.D.
1936	4,862	6,607	2.9	19.7	4,999	6,838	0.3	20.1
1937	4,989	6,677	3.1	19.0	5,448	7,247	0.1	19.5
1938	4,999	6,648	2.5	18.4	4,891	6,752	0.2	17.3
1939	1,499	3,132	0.4	26.0	1,545	3,336	0.0	30.2
1940	1,681	3,319	0.5	20.1	1,666	3,469	0.0	20.6
1941	1,496	3,341	0.2	22.8	1,929	3,795	0.0	27.7
1942	1,528	3,269	0.4	23.6	1,621	3,429	0.0	21.5
1943	1,500	3,472	0.4	24.4	2,014	3,945	0.0	24.0

Because of the cessation of all construction, an allowed diversion of 3,500 to 5,000 cu ft per sec for the duration of the war is the only measure that is immediately available to alleviate the existing conditions in the Main Channel and Brandon Pool. Such a flow would also be helpful in the lower Illinois River below Peoria, Ill., and Pekin, Ill., where greater activity in the manufacture of alcohol from grain has increased the pollution in the river by an

equivalent population of about 750,000 to 1,000,000. As a result, during the summer of 1943, the river was reported almost devoid of dissolved oxygen for a distance of 46 miles below Peoria.

Incidentally, it is of interest to note that the level of Lake Michigan has been rising since 1934 and, in August, 1943, averaged 581.47 above mean tide at New York, N. Y., as compared with a previous high monthly average of 583.49 in July, 1876.

In March, 1944, Congress had before it a resolution introduced by the Hon. A. J. Sabath, Representative from Illinois, to allow a diversion of 5,000 cu ft per sec. Hearings have been held before the Rivers and Harbors Committee of the House of Representatives.

From an entirely different angle, the Attorney General of Illinois, George F. Barrett, recently petitioned the United States Supreme Court, on behalf of Illinois, to enjoin the State of Indiana and the cities of East Chicago, Gary, Hammond, and Whiting from polluting Lake Michigan, the source of the Chicago water supply. The Court refused to dismiss the petition and on March 7, 1944, appointed a Special Master, Luther E. Smith. The polluted condition of the lake and the Grand Calumet River in Indiana has been a source of worry to sanitarians since 1909.

The moral of this story has been pointed by the poet, Robert Burns, in his saying:

"The best laid plans o' mice and men
Gang aft a-gley."

Certainly, periods of panic and depression and war stress have wrecked the prophecies of optimists and upset the plans for a rational program of construction and operation.

In his testimony, Mr. Buswell stated definitely that there could be odors even if nitrates were present (Record, page 1894). He also stated that "sludge banks, therefore, may prove offensive even though overlaid by a liquid containing some dissolved oxygen" and that "the Drainage Canal would become deoxygenated just as quickly if nitrates were present as if they were absent." His statement, in the discussion of this paper, that "It is important to note that all agreed that chlorination was an effective means for odor control" should be qualified since the witnesses for Illinois limited their opinion on the value of chlorine for odor control to sewage treatment works.

Mr. Howson states that "some of the procedures eventually incorporated into the Sanitary District's program were first urged by the experts for the Opposing States." The engineering staff of the Sanitary District, both regular and consulting, was well aware of the various alternatives available, but did not attempt to decide which was best, under the stress of litigation. Naturally, the staff was gratified to know that the experts for the Opposing States would not oppose certain procedures. The late Harrison P. Eddy, Past-President, Am. Soc. C. E., the late George W. Fuller and the late T. Chalkley Hatton, Members, Am. Soc. C. E., had recommended lagooning of excess sludge in 1923, antedating Mr. Howson's expression of the same idea as of 1941.

Mr. Howson compares the 1927–1928 estimates for a sewage treatment program on the basis of $176,000,000 for the Sanitary District and $76,000,000 to $82,250,000 for the lake states. The record clearly shows that such estimates are not comparable since the lake states omitted more than $46,800,000 of work, including intercepting sewers and pumping stations essential to operation. This is manifestly an incomplete program.

The intercepting sewer system is largely completed (except for the South Side and miscellaneous sewers), the North Side works are practically completed, and the Calumet works nearly completed. However, the Southwest and West Side works are not completed in the sense inferred by Mr. Howson. Lack of funds postponed vital work, so that the efficiency in removal of B.O.D. of the four major plants has averaged about 60% for 1941–1943, instead of 90% or better.

The expenditures for sewage treatment have been as follows:

Period	Actual expenditure
Prior to December 31, 1928 (as reported to U. S. Supreme Court, First Report)	$ 81,917,602.83
December 31, 1928, to December 31, 1938 (as reported to U. S. Supreme Court, Final Report, December 1, 1938)	77,877,728.87
December 31, 1938, to December 31, 1943	11,705,000.00±
Total to December 31, 1943	$171,500,000.00±

The estimated cost of the remaining work is as follows:

Types	Estimated cost
Essential (including completion of Southwest works and addition of activated sludge plant to West Side)	$ 10,500,000.00
Postponed (including South Side intercepting sewers and miscellaneous sewers)	22,940,000.00
Total	$ 33,440,000.00

Contrary to Mr. Howson who states that "Remedial Measure No. 9 'budgeting of the diversion' was also the initial suggestion of the experts for the lake states," budgeting had already been practiced by the Sanitary District as shown by Mr. Ramey's discussion.

The outcome of the litigation was happy in that certain very experimental temporary measures were vetoed by the Master, who favored the permanent work. The writer agrees heartily with the two closing paragraphs of the discussion by Mr. Johnson. To those who have contributed to the discussion the writer is grateful.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES

Discussion

By A. A. Eremin, and Richard M. Hodges

A. A. Eremin,[38] Assoc. M. Am. Soc. C. E.[38a]—An interesting method of analyzing the stresses in skewed rigid frames has been demonstrated in this paper. The tabulation of the equations, as in Table 6, serves to clarify the

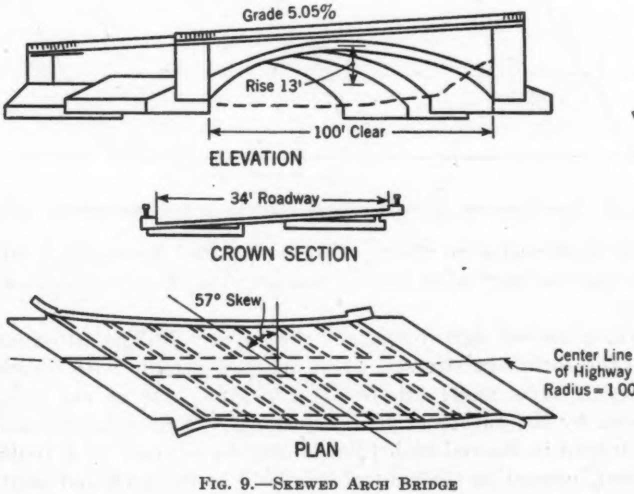


Fig. 9.—Skewed Arch Bridge

procedure and results in a saving of time and labor. Before this method can be used with complete confidence, however, further experimental study is needed. The equation for the torsion factor, F, cited by the author in his

Note.—This paper by Richard M. Hodges was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. C. D. Geisler, and Arthur G. Hayden; November, 1943, by Bernard L. Weiner, and Alfred L. Parmé; February, 1944, by J. Charles Rathbun; and March, 1944, by Maurice Barron, Phillips H. Lovering, and Jaroslav Polivka.

[38] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[38a] Received by the Secretary February 23, 1944.

definitions was originally developed by C. Bach for the purpose of computing stresses in reinforced concrete arch ribs. The ratio of breadth to depth in a concrete arch rib section is generally about 1 to 4, whereas in skewed rigid frames the ratio is often as great as 1 to 20.

Loading on skewed rigid frames or arches often consists of concentrated forces eccentrically placed in relation to the longitudinal center line of structure. Such forces produce increased local stresses that are not considered in the equations of this paper.

In detailing skewed bridges the limitations of the torsion factor and local stresses may often be minimized conveniently. Consider, for example, the Gaviota Bridge in California, constructed in 1932, which carries a heavy traffic load on the main highway between San Francisco and Los Angeles.[39] This bridge, with an unusually large skew angle of 57°, was constructed with two separate skewed arch ribs as shown in Fig. 9. Therefore, the breadth-depth ratio was reduced considerably. The roadway slab is supported on the transverse rigid walls with resulting uniform distribution of the concentrated forces.

Mr. Hodges has stated that, in analysis of stresses in skewed arch ribs, the direct deformations may be neglected. In skewed rigid frames the direct

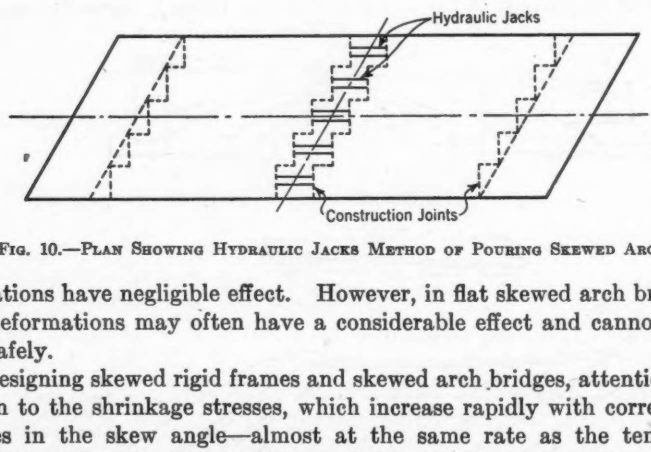


FIG. 10.—PLAN SHOWING HYDRAULIC JACKS METHOD OF POURING SKEWED ARCH

deformations have negligible effect. However, in flat skewed arch bridges the direct deformations may often have a considerable effect and cannot be neglected safely.

In designing skewed rigid frames and skewed arch bridges, attention should be given to the shrinkage stresses, which increase rapidly with corresponding increases in the skew angle—almost at the same rate as the temperature stresses shown by the author in his conclusion 3. A considerable part of the shrinkage stresses in skewed arch bridges may be reduced by a toothed construction joint, normal to the neutral axis, and in the arch and abutments,[40] as shown in Fig. 10.

RICHARD M. HODGES,[41] M. AM. SOC. C. E.[41a]—The writer is sincerely grateful to all who have gone to the trouble necessary to discuss this paper. In a few instances the comments seem to have particular reference to structures

[39] *Engineering News-Record*, March 30, 1933, p. 399.

[40] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1329, Fig. 38.

[41] Chf. Structural Designer, Gilmore D. Clarke, Cons. Engr., New York, N. Y.

[41a] Received by the Secretary February 16, 1944.

of such magnitude and cost that simplified methods would not be of much importance. It is to be hoped that all will lead, in one way or another, to a wider knowledge of what constitutes good practice in the analysis and design of concrete solid-barreled skewed frame and arch structures of moderate span.

The discussions plainly indicate the wide variation in the design methods now extant—from somewhat complicated attempts at theoretical precision at one extreme to rather rough approximations at the other. Perhaps it would be more correct to classify, at the latter extreme, the efforts being made to avoid the problem in various ways while still retaining the skew, such as the substitution of a ribbed structure for a solid-barreled structure, as mentioned in several of the discussions.

There is certainly something to be said in favor of attempting to make the design approach academic accuracy, in spite of resulting complications. (The writer is referring particularly to the effect on the design of the internal moments and stresses.) However, it is sometimes easy for those primarily interested in theory to overlook the practical elements of a problem. There is always a certain limit of justifiable precision in the practical design of a skewed frame or arch, or of any other structure. That limit is overreached when the degree of precision is such that (1) it cannot be warranted by the degree of accuracy attainable in the underlying data or in the basic assumptions; or (2) the cost of the design becomes unnecessarily disproportionate to the cost of the structure. There is even an element of danger in overprecision, in that it leads to a false sense of security. Another limiting factor of the utmost importance in connection with the design of moderate-span skewed frames and arches is established by the fact that, the more difficult or abstruse the design method, either to understand or to apply, the greater becomes the tendency of designers to go to the opposite extreme and to use inadequate approximations or rule-of-thumb methods, or even to shun this type of construction entirely. Of course any exact determination of any of the limiting conditions referred to is practically impossible—the least one should do about them, however, is to keep them in mind.

In several of the discussions, notably in those of Messrs. Parmé, Polivka, and Weiner, there has been a tendency to overlook the purpose and scope of this paper, which is merely an attempt, based on existing theory, to present a method of design, which, for the type of structure considered, will make it possible to meet the foregoing general requirements.

Mr. Geisler's discussion presents some interesting ideas. The tendency, to which he refers, of the forces to concentrate at the corners is shown graphically in Fig. 6. However, the effect on the longitudinal stresses is believed to be practically negligible, and, as stated by Mr. Geisler, there have been no indications of resulting distress in the numerous bridges designed and constructed without considering this effect.

It is common practice to construct the barrel with one or more construction joints parallel to the spandrel walls, as may be necessary, but the writer sees no advantage in dividing the barrel into comparatively narrow longitudinal

strips, whether or not more or less effectively tied together. The only way to get rid of the increased corner reactions, as well as of other skew effects, would be to make the separations complete and effective all the way from the crown down to the bottoms of the vertical legs, and also to remove the earth pressure by some kind of open deck approach construction. The effect of Mr. Geisler's method seems to approximate the usual solid-barreled type of construction, with the transverse ties tending to perform the function of transverse reinforcement as designed by the method here adopted.

It is quite true that the corner reaction intensities increase with the width of the structure, but the effect is ordinarily not very serious. However, they should not be neglected entirely, as is too frequently the case. The increase in the vertical reactions at the obtuse corners is relieved by yielding, as Mr. Geisler states, when the structure is founded on compressible soil, and in such a case the bridge itself must resist the additional torsion. If the bridge is designed to withstand the extra torsion, as the writer has recommended, by assuming free rotation about the x-axes, there is no cause for concern. Incidentally, for severe skews extreme temperature rise leads to a reversal, and then the vertical reactions will increase toward the acute corners. A similar reversal also occurs in the normal direction of the lateral thrust along each abutment. These effects can become very important for a severely skewed frame founded on piles.

Mr. Parmé, as well as Mr. Geisler, mentions the tests made by the Bureau of Public Roads.[8] These tests were made on a model with one abutment hinged and the other fixed. The Society's Special Committee on Concrete and Reinforced Concrete[42] stated that "The results of these tests could not be used conveniently as a check of Professor Rathbun's theory." In fact, the tests were not of a skewed arch with either fixed or hinged abutments. This would seem to account for the discrepancies shown by Mr. Parmé. The tentative conclusions from these tests, as quoted by Mr. Geisler, may still be true (they agree in general with the writer's investigations), but no interpretation that could be derived from them would invalidate the elastic theory developed by Professor Rathbun.

In Mr. Parmé's discussion, and also in that of Mr. Polivka, the basic theory for finding the resultant reactions has been questioned in spite of the fact that its essential accuracy was confirmed, in 1926, by a comparison between theoretical analyses by Professor Rathbun and model analyses by the late Professor Beggs. Mr. Parmé's analysis appears interesting, but the preponderance of opinion seems to be rather solidly against his and Mr. Polivka's evaluation of previous investigations. Since Mr. Parmé attempts to show that Professor Rathbun's theory is incorrect, and, since Mr. Polivka seems to think that nothing that has been done to date is worth considering, the writer considered it proper that Professor Rathbun should be afforded an opportunity to present his views. Accordingly, in a letter addressed to the writer, he states:

[8] "Progress Report on Skew Arch Tests," by G. W. Davis, *Public Roads*, Bureau of Public Roads, U.S.D.A., Vol. 6, No. 9, November, 1925, p. 185.

[42] "Concrete and Reinforced Concrete Arches," Final Rept. of the Special Committee, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1552.

> "*Transactions* [Vol. LXXXVII] 1924, p. 676 (3) covers the case analysed mathematically by Mr. Parmé and shows that his results are in 'sharp contradiction to established fact.' He will find, if he follows his analysis through all the terms, that he will arrive at the same results as I did, unless I misunderstand his rather short outline of his analysis."

Mr. Parmé cites but two experiments. One supports the elastic theory and the other does not. Four others that do support the theory are described elsewhere.[22, 43, 44, 45] Mr. Polivka cites no authority, but the foregoing also answers his statement that the existing method of analysis "* * * has not yet been corroborated satisfactorily by tests on models * * *."

The writer feels greatly complimented in having his paper approved, in general, by Mr. Hayden. An interesting feature of Mr. Hayden's discussion is his short history of the development of skewed arch and rigid frame design. The writer will take the liberty, when replying to Mr. Weiner, of filling in a few details which Mr. Hayden has omitted.

Mr. Hayden's idea (also mentioned by Professor Rathbun) of using the skew span, instead of the square span, as a basis of deriving the simplification, is well worth investigating. However, the writer thinks that it is more logical to develop a simplification along the lines of the universally applicable elastic theory than to attempt to use the Rankine approximation.

Both Mr. Hayden and Professor Rathbun have called attention to the common fallacy that skew effects can be avoided by the use of ribbed construction. Many bridges have been designed and constructed in this happy belief, and no trouble has resulted, to the writer's knowledge, but nevertheless these bridges are incompletely designed, and headroom has been needlessly wasted in the process.

A more serious matter is the practice of using approximations without knowledge or discrimination. The writer has read several publications (although not very recently), which stated that the longitudinal reinforcement of skewed frames can be designed safely by multiplying the square span moments and thrusts by the square of the secant of the skew angle, without considering that temperature moments and thrusts (which are the controlling factors in proportioning a frame of moderately heavy skew) must be multiplied by the fourth power of the secant.

The responsibility for this type of advice can be placed squarely on those engineers who, knowing something about skewed arch and rigid frame design, not only have paid no attention to the necessity of making it simple enough for practical use in the design of small frame structures, but have seemed actually to resent any effort that may be made in that direction by any one else.

Mr. Weiner's discussion takes the form of a dissertation on the subject of skewed arches in general, in the course of which he often ignores the scope

[22] "Crown Stresses in a Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

[43] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *ibid.*, Vol. 98 (1933), p. 26.

[44] *Ibid.*, p. 50.

[45] *Ibid.*, p. 66.

of this paper unnecessarily and takes frequent occasion to discuss his own paper[6] published in 1931. For instance, he states:

> "Much confusion indeed has been caused by the constant search for new 'methods.' * * * The writer has given complete equations[6] for both the symmetrical and unsymmetrical skew arch (single span) with the origin and basic structure chosen with this [simplification of equations] idea in mind."

and, subsequently:

> "* * * it was partly because the original skew-arch [Rathbun] theory was written on the basis of a space system that it had to be revised for efficient use in the drafting room, and this revision constitutes Part I of the writer's paper[16]—and is so stated therein."

It is significant that no mention of this revision by Mr. Weiner is made in the "Historical Perspective" part of Mr. Hayden's discussion. It was because Mr. Weiner's revision proved too laborious (except to Mr. Weiner and the men working in his squad) that it was considered necessary to make the subsequent adaptation of the original Rathbun method to which Mr. Hayden refers.

Mr. Weiner has brought up a number of points which are not directly related to the current paper but consist of criticisms of Professor Rathbun's previous contributions. In reply to an inquiry by the writer, Professor Rathbun states that his interest is purely in

> "* * * the development of the skew arch theory and its acceptance and use by the Profession. Mr. Weiner has written extensively on the skew arch. He stated that my 'theory [page 1330, 1932 *Transactions*] formed the basis of Part I of the writer's paper.' Mr. Weiner did revise the sign system and he changed the order of my equations. These were revisions which I consider unwise since they lead to confusion. He also introduced the idea of the ellipse of stress. As I have pointed out in previous discussions of papers, this idea is incorrect as here applied. To my knowledge Mr. Weiner has made no other contribution."

The writer is opposed to Mr. Weiner's ellipsoidal stress method for the additional reason that, even if it were theoretically correct in its application, it would still be unnecessarily complicated as a matter of practical design.

Opportunities to discuss the relative technical merits of Part II of Mr. Weiner's paper and of the design method originally proposed by Professor Rathbun were offered in their proper place and need not be extended in this closing discussion. The writer will not attempt to answer all of Mr. Weiner's numerous criticisms of this paper. He will answer the statements made in Mr. Weiner's opening paragraphs and will then attempt to use a sampling method on the others.

The first paragraph, under his side heading, "Step I. Reactions," is incorrect throughout. Mr. Weiner states,

> "The purpose of this part [computation of reactions] of the paper is to show that permissible approximations may be made in the frame con-

[6] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1212.

[16] *Ibid.*, p. 1214.

> stants with the result that the equations developed on the basis of the accepted theory (proved by tests) are simplified. In addition, the equations break up into groups, each of course containing fewer unknowns. That permissible approximations usually result in simplifications is, of course, obvious; nevertheless, the results are approximate and the paper has been misnamed. Neither proof nor data are given for the degree of approximation."

This paper is not concerned in the least with "permissible approximations" in "frame constants"; and it is misleading, at least, to state that the equations referred to are simplified. The "equations referred to" were not used at all. They were not simplified. New equations were derived on a simplified elastic basis.

Mr. Weiner has only described the process used by himself (and before him by Professor Rathbun) in arriving at approximate formulas for certain redundant reactions. These approximate formulas were derived simply by applying the original equations to a skewed frame with a perfectly flat top and vertical legs. When the writer first began to search for a reliable method of simplification, he tried this method and found it inadequate. Certain reaction components depending largely on deck curvature could not even be approximated roughly in this way. Others were close within certain limits of skew and curvature; but these limits were unknown. The only approximation that stood the test of comparative analyses was the well-known formula "$R_x = H$" (using the writer's notation), which was first published, not by Mr. Weiner, but by Professor Rathbun. In the "Appendix," the writer has given (as a matter of interest and not as an original contribution of theory) a derivation of this important relationship, as adapted to the system of coordinates used in this paper, showing the assumptions on which it is based. The method described by Mr. Weiner was found inadequate for obtaining the other redundant reactions. Instead of simplifying the existing equations as adapted from the original elastic theory, the writer derived new equations for Part 2, using only one instead of two elastic systems. There is no resemblance between the process used by the writer under the heading, "Derivation of Method for Determining Reactions," and the process attributed to him by Mr. Weiner.

The statements made in the last few lines of the paragraph previously quoted cannot be ignored. Before preparing this paper, the writer had obtained evidence of the closeness of the fundamental relationship that $R_x = H$ from comparative analyses of different frames of varying types. The writer stated this very clearly, and he had hoped that it would not be necessary to burden the reader with detailed descriptions and summaries of these comparisons, but Mr. Weiner has left no alternative. (Attention is called to Mr. Barron's discussion of this point, which represents a more constructive and useful form of skepticism than that of Mr. Weiner.)

The following summary is obtained from the results of comparative test analyses made in 1932 by the office force headed by Mr. Hayden, then Designing Engineer of the Westchester County Park Commission, under the direct supervision of the writer. Analyses were made of two bridges, representing different types of rigid frame construction. The Main Street Bridge, Central Westchester Parkway, White Plains, N. Y., is a rigid frame of average

type (span 50 ft and rise 21 ft) with a comparatively small degree of deck curvature. The Loop Bridge, carrying the Bronx River Parkway Extension over the Taconic Parkway Connection, is an elliptical rigid frame, approaching a semicircular full-centered arch (clear span 42.5 ft and rise 23 ft). Both were constructed in 1931 and in both bridges the angle of skew was 38°.

Comparative analyses were made for various skew angles. For skews of more than zero, all reaction components were computed by Professor Rathbun's theory as adapted for the Park Commission. For skew angles of 0°, the reaction components R_x (or H) were computed by the ordinary elastic theory for two-hinged rectangular frames or arches. Calculations were made for a temperature rise of 50° F, for balanced earth pressure, and for unit vertical loads applied successively at three points which may be described approximately as near the haunch, quarter point, and crown. The results as given in Table 14

TABLE 14.—SUMMARIES, VALUES OF R_x

Skew angle	Main Street Bridge					Loop Bridge (Elliptical)				
	Unit Load at			Earth pressure	Temperature	Unit Load at			Earth pressure	Temperature
	Haunch	¼ point	Crown			Haunch	¼ point	Crown		
0°	0.201	0.383	0.495	1.80	0.156	0.060	0.220	0.330	2.38	0.068
38°	0.202	0.384	0.499	1.75	0.158	0.059	0.221	0.332	2.33	0.072
45°	0.201	0.381	0.497	1.73	0.154	0.059	0.221	0.332	2.31	0.072
60°	0.202	0.376	0.510	1.76	0.154	0.058	0.221	0.337	2.24	0.072

are for a unit width. The customary temperature corrections were applied in reverse, and the skew span reaction components were divided (instead of multiplied) in this case by $\sec^2 \theta$ to make them comparable with rectangular frame reactions.

Near the beginning of his discussion, Mr. Weiner states:

> "Many years ago the writer found that for a symmetrical structure the approximations were permissible and result in the simplifications stated."

Presumably Mr. Weiner is referring to the simplifications stated in this paper. The only approximation used in the paper was found by Professor Rathbun. The others are not permissible for a symmetrical structure except under certain restrictions of small deck curvature and skew. The results of Mr. Weiner's analysis, as well as the methods used, are totally different from those advanced in this paper. According to Mr. Weiner (and Professor Rathbun before him), $R_z = \epsilon R_x$, using the writer's notation. This is a fairly close approximation for restricted deck curvature and skew. According to the method presented in this paper, $R_z = \epsilon R_x + R'_z$, in which ϵR_x is the static equilibrant of R_x, and R'_z is that part of R_z which is due to the torsional elasticity of the structure. Since M_x and M_y are both functions of R'_z, and since R'_z does not appear in Mr. Weiner's approximate formulas, Mr. Weiner's statement is incorrect.

Mr. Weiner favors the location of the origin of the coordinate system at the crown, rather at one of the hinges, which is permissible, but the writer disagrees emphatically with the contention that this is more logical for a hinged structure. For those who are accustomed to designing rectangular frames with the origin at a hinge, the hinge is the logical point of origin for the design of hinged skewed frames. One advantage of this arrangement is that the sign system is much simplified.

Some of Mr. Weiner's comments, regrettably, serve only to introduce complications and confusion. For example, the method under discussion needs no elaborate sign system. No one has ever considered the introduction of such a system in the design of skewed frames as finally adapted for the Westchester County Park Commission, on which the method under discussion is based. It would have been unnecessary, and that is one of the chief advantages of this method over Mr. Weiner's adaptation. The best sign system in any case is the one that requires the very least in the way of "formal" explanation. Mr. Weiner states that the writer "does not give a formal statement as to the directions of the positive moments and thrusts." The statements made in the paper (see "Derivation of Method for Determining Reactions: Details of Derivation," fifth paragraph) are perhaps not as formal as they might be, but, in conjunction with Fig. 1, to which they refer, they make the sign situation very clear. Mr. Weiner states that "a 'picture' is not sufficient," which is true, but a clear picture, together with a simple explanation, is certainly the best way to avoid misinterpretation. (There is an oriental proverb that states, in effect, that "a picture is worth ten thousand words.") This is just as correct for multiple spans and multiple stories (which were not open to discussion) as it is for the simple one span frame which is the subject of this paper. Definitions can mean various things to various people, but a picture always means the same to every one.

The system of setting up formulas for each step in a design process, and having "juniors" perform blind substitutions, may be temporarily efficient under certain conditions, although pretty hard on the "juniors." It is certainly a poor way to develop future designers. The writer is convinced that this procedure would be inadvisable as a general policy for any engineer or any organization.

Mr. Weiner's comment on one of the recommendations in the paper is that

> "The writer [Mr. Weiner] also fails to understand the reasoning on the earth-pressure distribution. Once equivalent fluid pressure is assumed, it acts normal to the surface of the wall and that is all that there seems to be to it. It is true, of course, that the equivalent fluid pressure theory leaves much to be desired in the way of a rational theory, but this rather unfortunate situation is not unique with the skew arch; it is true of all structures subjected to earth pressure."

This statement indicates a misinterpretation of the recommendation. There is no "unfortunate situation" involved, and the writer's recommendation certainly does not apply to anything but skewed bridges.

Unbalanced earth pressure is rarely used in the design of symmetrical frames under ordinary conditions. With proper specifications and proper inspection

(which the designer has a right to expect), the chance of such a condition is too remote to justify the extra time and trouble. There are so many combinations of active, passive, and unbalanced earth pressure and restricted sidesway conditions to be considered, that there would be no end to the time that might be spent on this kind of investigation. Failure to take advantage of the neutralizing effect of balanced earth pressure is failure to benefit by one of the principal intrinsic economies of the rigid frame type of construction.

The writer does not agree that vertical concentrated load tests supply the information obtainable from earth-pressure tests and temperature tests. Vertical loads placed in various positions would be of little value in deciding, for instance, how much (if any) of the lateral earth-pressure reactions is actually carried along the abutments in friction, or whether it is actually true, as far as temperature is concerned, that the structure can be made stronger by using inferior concrete.

Mr. Weiner asserts that the writer "quotes Professor Rathbun incorrectly when he states that the transverse shearing force is distributed parabolically * * *." The writer did not quote Professor Rathbun, nor did he make any such statement. What the writer did state was that "the direct transverse shear is assumed to be distributed parabolically * * *," and the quotation is in fact from Mr. Hayden's book. Mr. Weiner's error is in overlooking the phrase "is assumed to be." This assumption (which was used for many years before this paper was written) is one of several in Part 3 of the design which have led to modifications in the original method. The purpose has been to simplify the design computations further, and also to increase the margin of safety in proportioning the transverse reinforcement, thus making allowance for the uncertainties (frequently referred to in the paper) affecting this part of the design.

In "General Remarks," Mr. Weiner states that designing by the 1-ft strip method "is merely a minor arithmetic convenience which has no structural significance." The same can be said of any short cut. Such a statement may even apply to a slide rule or a calculating machine, and certainly any convenience that saves time and eliminates possible errors is justifiable.

The writer hopes, with Professor Rathbun, that this paper will renew interest in the subject and perhaps be the means of inducing designers to familiarize themselves with the basic theory. It is only by such means that, among other advances, improved simplifications for practical design purposes can be evolved.

The writer is particularly gratified by the contribution of Mr. Barron, who immediately understood that the fundamental basis of the method under discussion is predicated on a complete separation of the two elastic systems which have always previously been considered together, and then proceeded to explore the subject on this basis. He not only found that the rectangular reaction components R_x are independent of the skew angles in the case of the two elliptical bridges which he analyzed, but also gave a general demonstration of why this condition holds true for every rigid frame investigated. The elastic-center, elastic-weight method employed by Mr. Barron reveals this fact much better than the method of analysis customarily used by the writer.

Mr. Barron has also made a very apt comparison between the method under discussion and the usual practice of neglecting shear deformations, and, in most cases, rib shortening, in the design of ordinary rectangular frames and arches. The writer is in agreement with this statement. He has not claimed that the basic relation is mathematically exact, although it proved so close in the case of some of his comparisons that he was almost tempted to believe so.

Mr. Barron speaks from experience when he mentions the difficulties encountered in considering the flexural and torsional elastic systems together in one group of equations. In such case, the same difficulties and small differences are always encountered, no matter what method is employed.

The writer agrees that rotation about the "y-axes" is highly improbable. However, there have been cases of separate footings for abutments and approach walls, and very little complication is added if such rotation is considered as a possibility. Incidentally, Mr. Barron is the only discusser who has paid much attention to the special problem of earth-pressure loading.

Mr. Lovering's discussion is short, but much to the point in discussing the question on which the practical value of the writer's method entirely depends,—namely, how does it work? The discussion emphasizes clearly the fact that skewed frame designs cannot wait for the solution of theoretical niceties and academic abstractions. They must be turned out quickly to meet present requirements, if they are to be used to any appreciable extent. Professor Rathbun showed that he realized this many years ago when he originated the very simple and straightforward method of practical design for these structures which has been used by the writer, with certain modifications for the sake of further simplicity. Mr. Lovering has stated the practical advantage of being able to establish the proportions of the structure once and for all before considering the somewhat more complicated torsional effects that apply only to the subordinate part of the analysis and design. He has emphasized the fact that temperature is the controlling factor in the design of all rigid frames of considerable skew, and has shown a very practical and economical way of minimizing the temperature effects in such cases, making it possible to extend the practical skew limits of any particular skewed frame or arch. He has also recommended an effective way of transferring the shears between the legs and the footings which should be standard practice. The writer had hoped that there would be more discussions of the type submitted by Mr. Lovering, dealing with the more practical and more neglected aspects of the problem.

Mr. Eremin's reference to the unreliability of the torsion factor calls attention again to the fact that there is no such thing, at present, as an exact method of design for the torsional effects on a skewed frame or arch structure. This fact has been previously referred to as a good reason for avoiding all misleading methods of unwarranted precision. It has also been mentioned that torsional effects are not neutralized by ribbed construction or by dividing the arch or frame barrel into separate strips, and that they can be taken care of safely and without undue loss of economy by proportioning the transverse reinforcement on the basis of extreme assumptions for this subordinate part of the design. As Mr. Lovering has stated, the controlling design, from which

the structure itself and its longitudinal reinforcement are proportioned, can be made independently. It is not affected in any way by such assumptions.

The problem of local stresses caused by eccentric loading is an uncertainty which applies to rectangular as well as to skewed solid-barreled structures. The writer questions the necessity of any expedients either in the design or construction of solid-barreled skewed frames or arches. Mr. Eremin calls attention to the necessity of considering rib shortening stresses, which multiply themselves in skewed structures in the same way as temperature stresses, in the design of skewed flat arches. This comment should also apply to skewed frames of exceptionally long span.

Since the inception of this paper, it has become increasingly apparent that, with the modern trend toward divided traffic arteries, double span rigid frames may soon become the rule and single spans the exception. The rather common practice of designing such double span bridges by methods similar to Part 1 of this paper and then more or less guessing at the transverse reinforcement is probably a reasonably safe procedure in most cases; but it is evidently not a very reliable or satisfactory one, particularly in regard to the transverse reinforcement. Although the first part of the aforementioned procedure may be correct enough for practical purposes, an attempt should be made to establish conclusively by comparative analysis (as a further check on Mr. Barron's reasoning in the last paragraph of his discussion) the fact that it is proper to separate the two elastic systems in the same way as for a single span structure. If this is true, as expected, it will be possible to derive comparatively simple equations for the torsional and shear redundants, especially for two equal spans, and to arrive at a reliable method of completing the design with much less work than is now required.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Discussion

BY NEIL VAN EENAM, AND C. H. GRONQUIST

NEIL VAN EENAM,[47] M. AM. SOC. C. E.[47a]—The problems of research, design, fabrication, and erection involved in the construction of the new Rainbow Arch Bridge are presented in a clear and thorough manner in this Symposium. Until recent years, no solid rib arch would have been structurally and economically feasible, or even possible, for the 950-ft span over the Niagara gorge, because of the excessively thick web plates that would have been necessary. Now, methods are available for stiffening the web plates longitudinally so that plates of reasonable thickness may be employed. On the Rainbow Arch Bridge, longitudinal stiffeners have been used to the best advantage by making them serve an architectural as well as a structural purpose.

The investigation of deflections and of secondary stresses is an outstanding example of the thoroughness of the design. The authors deserve great credit for calling attention to the inadequacy of the elastic theory when deflections are appreciable. This word of caution in regard to the magnitude of the secondary stresses applies with equal force to the design of arch ribs of moderate span, particularly when two-hinged or three-hinged ribs are to be of shallow depth.

The Rainbow Arch Bridge is pleasing and graceful in appearance, and, because of the absence of bracing, it offers only slight obstruction to the view. If the rigidity of the spandrel columns is such that the lateral bracing may be omitted safely, the engineers have succeeded remarkably in building a monumental structure that harmonizes with the landscape.

NOTE.—This Symposium was published in October, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1943, by Egidio O. Di Genova, and Charles Mackintosh; January, 1944, by C. M. Goodrich, and T. Kennard Thomson; February, 1944, by C. M. Spofford, L. J. Mensch, and O. H. Ammann; and March, 1944, by Louis Balog, William G. Rapp, and Leon Beskin.

[47] Highway Bridge Engr., U. S. Public Roads Administration, Washington, D. C.

[47a] Received by the Secretary February 21, 1944.

C. H. GRONQUIST,[48] ASSOC. M. AM. SOC. C. E.[48a]—Only at rare intervals does there come to the bridge engineer a problem so satisfying to the engineering mind and heart as the bridging of the Niagara gorge. That the designers and builders of the Rainbow Arch Bridge have produced a solution which is eminently right is evident. The papers in this Symposium present more than adequately the record of this achievement of the authors.

A project which makes similar appeal to the bridge engineer is the Henry Hudson Bridge at Spuyten Duyvil in New York, N. Y. This 800-ft arch was built in 1936 as a four-lane bridge, and prior to the construction of the Rainbow Arch Bridge held the distinction of being the longest steel fixed arch bridge. The general designs of the two bridges are essentially alike, although an upper deck accommodating three lanes was added to the Henry Hudson Bridge in 1938 to transform it into a six-lane bridge, and there are many differences in important features of detail design. Although the design and erection of the Henry Hudson Bridge have been described elsewhere,[49,50] a brief comparison of the two structures should be included in the discussion of this Symposium.

The rib of the Henry Hudson Bridge, like that of the Rainbow Arch Bridge, consists of a silicon steel box girder of constant depth, except for a short section adjacent to each skewback. Unlike that of the Rainbow Arch Bridge, the rib section of the Henry Hudson Bridge (which is 12.6 ft deep and weighs 2,400 lb per ft) is composed of two separate girders 3 ft 8 in. center to center, connected by heavy lacing at both flanges and at mid-height.

The skewback section of the rib and the anchorage to the abutment are generally similar for the two bridges, and flexible connections were used at the top and bottom of the spandrel columns for both bridges to reduce temperature and participation stresses in the columns and deck framing. Lateral bracing between the spandrel columns in the Henry Hudson Bridge is laid out in conformance with the bracing in the steel towers surmounting the skewbacks and the bents of the approach spans, which were of steel construction for reasons of economy.

The Henry Hudson Bridge was erected simultaneously from both ends of the span by cantilevering over steel falsework bents supported by cage-braced steel piles driven to rock. The falsework bents were located near the eighth points and quarter points of the arch, and at the quarter-point bents the ribs were reinforced by toggles consisting of eyebar chains and a bent which later became the end spandrel bent of the bridge. The grillages were erected on steel supports set on the concrete of the skewbacks in such manner as to permit jacking the grillages in any direction to secure their accurate location.

Careful check was made of the actual distance between grillages before erecting the skewbacks, and from these data, and the length of the shop rib assemblies, it was determined that the crown closing piece for the Henry Hudson Bridge, nominally 30 in. long, should be fabricated $\frac{1}{2}$ in. short. A practically perfect check of the elevation of the crown of the arch after closure con-

[48] Associate Engr., Robinson & Steinman, New York, N. Y.

[48a] Received by the Secretary March 13, 1944.

[49] "Designing the Longest Fixed Steel Arch," by D. B. Steinman and C. H. Gronquist, *Engineering News-Record*, August 13, 1936, p. 232.

[50] "Special Problems of Hingeless Arch Erection," by W. K. Greene, *ibid.*, November 12, 1936, p. 669.

firmed the efficiency of the closure scheme. This is made further evident by considering that 1-in. error in length of the rib of the Henry Hudson Bridge (1-in. error in length of the span was assumed in the design), corresponding to 15° F change in temperature, would have changed the elevation of the crown 1.5 in., whereas the change in thrust would have been 28 kips, or only slightly more than 1% of the thrust at the time of closure. From the foregoing it is apparent that check of elevation of the axis is at the same time the most sensitive as well as the most simple check on the calculated stress in the rib at closure, and that, if proper precision is used in field layout and shop fabrication, no adjustment need be made at closure. The latter is essentially the conclusion of the authors of the third Symposium paper, to which the writer would take exception only by stating that if the span between grillages is known to be correct and if it is determined by measurement of length and elevation of rib at closure that the steel is not of correct length, such error should be compensated for by adjustment at closure.

Jacking arrangements at the crown were similar for the two bridges, except that for the Henry Hudson Bridge only one 500-ton jack was used at each flange, the width of opening between the half arches was 30 in., and a small pin was introduced in the bottom flange, where, on release of the jacking force, contact between the two half ribs would be made first. The crown jacks, which were interconnected and operated from a single pump to maintain non-eccentric axial load until initial closure was made, were first brought to capacity load. Because closure was made on a July day when the temperature rose to 90° F, it also was necessary to jack at the quarter-point bents, while maintaining constant load on the crown jacks.

The problem of determining the effect of deflection on the stresses in a slender arch is unquestionably of prime importance. In an effort to circumvent the laborious cut-and-try procedure for obtaining the final moments and deflections termed "exact" by the authors of the first Symposium paper, the writer, some time before the publication of this Symposium, derived a formula for the so-called "deflection-theory" correction of the usual elastic-theory moment and deflection that can be applied as soon as the moment of inertia of the rib at the crown has been estimated.

The usual assumption is made that the arch axis is parabolic and that the moment of inertia of the rib varies with the secant of the slope of the axis. Vertical deflections may then be computed by considering the rib as a horizontal beam of constant moment of inertia equal to that at the crown. Half-span live load on such a two-hinged arch produces deflection in two segments with zero moment and deflection at the crown. Neglecting the effect of distortion and shortening of the rib, the vertical deflection at either quarter point will be:

$$\eta_e = \frac{\dfrac{p\,L^2}{64}}{4 \times 9.6\,\dfrac{E\,I}{L^2}} = \frac{M_e}{P_{e-v}} \qquad (85)$$

The numerator of Eq. 85 is the value of the moment at the quarter point of

the arch. The denominator is the buckling load of the rib, for elastic curve deflection, taken as a horizontal strut of length $L/2$ between the end hinge and central point of contraflexure. Since there is zero deflection at the point of contraflexure, the half arch under this loading is analogous to a simple beam with a full-span uniformly distributed load. The center deflection for a fully loaded simple beam is also equal to the moment at the center divided by the buckling load of the beam, for elastic curve deflection, taken as a column of length L.

Written in terms of the quarter-point moment, the horizontal deflection at the loaded quarter point, neglecting the effect of distortion and shortening of the rib, is:

$$\rho_e = \frac{\dfrac{p L^2}{64}}{\dfrac{50}{41} \times 9.6 \dfrac{E I}{h L}} = \frac{M_e}{P_{e-h}} \qquad (86)$$

in which the denominator is again a buckling load, dependent on the rise h as well as on the span L of the arch.

For loading of this type, producing negligible deflection at the crown, the value of $H_w + H$ is practically unaffected by deflection, and the correction in moment due to vertical deflection is merely $(H_w + H)\,\eta$. To obtain the correction in moment due to horizontal deflection, it is assumed for simplicity that all points on the arch have the same horizontal deflection and that the position of the live load changes by that amount. Then at the quarter point the correction in moment is $\dfrac{\rho\,(w + p)\,L}{4}$. This assumption errs on the side of maximum deflection effect. The total final moment at the quarter point is:

$$M_f = M_e + (H_w + H)\,\eta + \frac{\rho\,(w + p)\,L}{4} \qquad (87)$$

If it is assumed that the deflection correction in deflection is in the same proportion as the deflection correction in moment, the expressions for final deflection become $\eta = \dfrac{M_f}{P_{e-v}}$ and $\rho = \dfrac{M_f}{P_{e-h}}$; or

$$\eta = \frac{M_e + \dfrac{\rho\,(w + p)\,L}{4}}{P_{f-v}} \quad \text{and} \quad \rho = \frac{M_e + (H_w + H)\,\eta}{P_{f-h}} \qquad (88)$$

in which

$$P_{f-v} = 4 \times 9.6 \frac{E I}{L^2} - (H_w + H) \text{ and } P_{f-h} = \frac{50}{41} \times 9.6 \frac{E I}{h L} - \frac{(w + p)\,L}{4} \qquad (89)$$

The final buckling loads as corrected for the effect of deflection are reduced from the values computed by the usual elastic theory by the amount of the horizontal thrust and the vertical shear term.

Solving Eqs. 88 simultaneously to obtain the values of the vertical and horizontal deflection at the quarter point:

$$\eta = \frac{M_e}{P_{e-v} \times \frac{P_{f-h}}{P_{e-h}} - (H_w + H)} \quad \text{and} \quad \rho = \frac{M_e}{P_{e-h} \times \frac{P_{f-v}}{P_{e-v}} - (w + p)\frac{L}{4}} \qquad (90)$$

Substituting Eqs. 90 in Eq. 87, the ratio of the final moment to that by the elastic theory is:

$$\frac{M_f}{M_e} = 1 + \frac{H_w + H}{P_{e-v} \times \frac{P_{f-h}}{P_{e-h}} - (H_w + H)} + \frac{(w + p)\frac{L}{4}}{P_{e-h} \times \frac{P_{f-v}}{P_{e-v}} - (w + p)\frac{L}{4}} \qquad (91)$$

If only the effect of vertical deflection is considered, Eq. 91 reduces to

$$\frac{M_f}{M_e} = 1 + \frac{H_w + H}{P_{f-v}} = 1 + \frac{H_w + H}{4 \times 9.6\frac{E\,I}{L^2} - (H_w + H)} \qquad (92)$$

With Eq. 92 written in terms of total thrust $(T_w + T)$ at the quarter point, and the buckling load computed for the length of the rib from the end hinge to the crown, the ratio of the final positive or negative moment and deflection to that obtained by the elastic theory, on the basis of radial deflections, is obtained. Thus:

$$\frac{M_f}{M_e} = 1 + \frac{T_w + T}{P_f} \qquad (93)$$

in which

$$P_f = \frac{4 \times 9.6\,E\,I}{L^2\left(1 + \frac{16\,h^2}{3\,L^2}\right)} - (T_w + T) \qquad (94)$$

Radial deflections are computed from the moment at the quarter point:

$$u_e = \frac{M_e}{P_e} \quad \text{and} \quad u = \frac{M_e}{P_f} = \frac{M_f}{P_e} \qquad (95)$$

The buckling load of Eq. 94 is the critical buckling load for the arch, neglecting the minor reduction due to the effect of curvature. This equation, therefore, should be used for stability computations in determining the factor of safety of an arch against buckling under all possible loads, as in Eqs. 16 and 17.

The foregoing relations have been previously developed for the stability of struts subjected to lateral loads,[51, 52] and their application to arches has been indicated.[43] Eqs. 14 will be found to reduce to the form of Eq. 93, since

[51] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

[52] "Airplane Structures," by A. S. Niles and J. S. Newell, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1943.

[43] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1936.

$P_e = \frac{M_{ae}}{u_{ae}}$; and also to the form of Eq. 92, if the horizontal thrust and vertical deflection only are substituted in Eqs. 14.

Applying Eqs. 91 to 95 to the two-hinged design for the Rainbow Arch Bridge for a half-span load of 1.5 kips per foot of rib:

$$\frac{M_f}{M_e} = 1 + \frac{7{,}350}{20{,}850 \times \frac{37{,}850}{40{,}350} - 7{,}350} + \frac{2{,}500}{40{,}350 \times \frac{13{,}500}{20{,}850} - 2{,}500} = 170.6\%;$$

$$\frac{M_f}{M_e} = 1 + \frac{7{,}750}{10{,}650} = 172.8\%; \qquad M_e = \frac{1.5 \times (950)^2}{64} = 21{,}150 \text{ ft-kips};$$

$$M_f = 1.728 \times 21{,}150 = 36{,}500 \text{ ft-kips};$$

$$\eta_e = \frac{21{,}150}{20{,}850} = 1.01 \text{ ft}; \qquad \eta = 1.706 \times 1.01 = 1.73 \text{ ft};$$

$$\rho_e = \frac{21{,}150}{40{,}350} = 0.525 \text{ ft}; \qquad \rho = 1.706 \times 0.525 = 0.90 \text{ ft};$$

$$u_e = \frac{21{,}150}{18{,}400} = 1.15 \text{ ft}; \qquad u = 1.728 \times 1.15 = 1.98 \text{ ft};$$

and $$[(\eta_e)^2 + (\rho_e)^2]^{0.5} = 1.14 \text{ ft}; \qquad (\eta^2 + \rho^2)^{0.5} = 1.95 \text{ ft}.$$

A satisfactory check is observed between deflection-theory corrections obtained on the basis of vertical and horizontal deflection components and radial deflection. The results are likewise in good agreement with those of the authors of the first Symposium paper by their approximate method. Considering only vertical deflections: $\frac{M_f}{M_e} = 1 + \frac{7{,}350}{13{,}500} = 154.5\%$. Therefore the effect of including the influence of horizontal deflection is to increase the deflection-theory correction from 54.5% to 70.6% or 72.8%.

Maximum positive moment by the elastic theory is produced at the quarter point for 0.43-span load rather than half-span load, and the horizontal distance to the point of contraflexure is 0.47 L rather than 0.5 L. Using this length in computing the buckling loads:

$$\frac{M_f}{M_e} = 1 + \frac{7{,}250}{23{,}400 \times \frac{40{,}230}{42{,}700} - 7{,}250} + \frac{2{,}470}{42{,}700 \times \frac{16{,}150}{23{,}400} - 2{,}470} = 158.2\%;$$

$$\frac{M_f}{M_e} = 1 + \frac{7{,}700}{12{,}900} = 159.6\%; \qquad M_e = \frac{1.5 \times (950)^2}{60} = 22{,}550 \text{ ft-kips};$$

and $M_f = 1.596 \times 22{,}550 = 36{,}000$ ft-kips. The final moment is little different for the two loadings, since the deflection correction is reduced with reduction in length to the point of contraflexure.

Although not discussed by the authors, maximum moment is produced at the crown by a central 0.3-span loading. Horizontal deflection of the haunches is small, and may be neglected in obtaining the deflection correction approximately. Deflection is in three segments or loops, with an accompanying increase in $H_w + H$ due to crown deflection. Since the distance between

points of contraflexure is reduced to about 0.33 L, the arch is stiffer than under half-span load, and the deflection correction is correspondingly smaller. The value of $H_w + H$ is changed by deflection; therefore the deflection-theory increase in moment due to vertical deflection cannot be obtained as for half-span load, but it can be approximated by application of Eq. 92, using the reduced value of 0.33 L in place of $L/2$ for the length of strut in computing the buckling load, and using the value of H computed by the elastic theory. For the Rainbow arch two-hinged design: $H_e = 0.0565 \times 1.5 \times \frac{(950)^2}{150} = 510$ kips; $P_{f-v} = \left(\frac{0.50}{0.33}\right)^2 \times 20{,}850 - (6{,}790 + 510) = 40{,}700$ kips; $\frac{M_f}{M_e} = 1 + \frac{7{,}300}{40{,}700} = 117.9\%$; $M_e = 0.0725 \times 1.5 \times (950)^2 = 9{,}800$ ft-kips; and $M_f = 1.179 \times 9{,}800 = 11{,}550$ ft-kips. This correction would be increased slightly by use of Eq. 92, with the buckling load computed for the length of rib between points of contraflexure to account for the effect of horizontal deflection.

The problem of approximating the deflection-theory correction for the fixed arch near the quarter point is more complex than for the two-hinged structure, since both linear and angular deflection occur at the point of contraflexure near the skewback of the former, whereas only angular deflection can occur at the end hinge of the latter. The value of $H_w + H$ is only slightly affected by deflection for either type, however, vertical crown deflection being negligible. The percentage of deflection-theory correction for the fixed arch, as for the two-hinged arch, is best computed by Eqs. 92 and 93, using the distance between the points of contraflexure for the loading considered. The percentage so computed is practically constant between the respective points of contraflexure, a considerable decrease occurring only near the skewbacks in the fixed arch.

Final deflections for both types of arches may be approximated by increasing elastic-theory deflections by the percentage of deflection-theory correction for moment. The maximum vertical deflection by the elastic theory at the five-sixteenth point of a fixed arch for half-span load may be written:

$$\eta_e = \frac{\dfrac{9\,p\,L^2}{1{,}024}}{\dfrac{192}{35} \times 9.6\,\dfrac{E\,I}{L^2}} = \frac{M_e}{(P_{e-v})'} \quad \text{(96)}$$

in which the numerator is the moment at the five-sixteenth point of the fixed arch, and the denominator is a buckling load. This buckling load is smaller than the buckling load for the 3/8 L distance between the points of contraflexure for half-span load, however, because the total vertical deflection, not the deflection from a straight line joining the points of contraflexure, is being measured. Similarly, the horizontal deflection at the five-sixteenth point, written in terms of the moment at the five-sixteenth point, is:

$$\rho_e = \frac{\dfrac{9\,p\,L^2}{1{,}024}}{\dfrac{384}{205} \times 9.6\,\dfrac{E\,I}{h\,L}} = \frac{M_e}{(P_{e-h})'} \quad \text{(97)}$$

For the Rainbow Arch Bridge with a half-span load of 1.5 kips per foot of rib: $P_{f-v} = \frac{64}{9} \times \frac{9.6 \times 29{,}400 \times 11{,}688}{(950)^2} - (5{,}910 + 560) = 19{,}530$ kips;

$$P_f = \frac{26{,}000}{1 + 3\left(\frac{150}{950}\right)^2} - 6{,}700 = 17{,}500 \text{ kips}; \quad \frac{M_f}{M_e} = 1 + \frac{6{,}470}{19{,}530} = 133.2\%,$$

considering vertical deflection; $\frac{M_f}{M_e} = 1 + \frac{6{,}700}{17{,}500} = 138.2\%$, considering radial deflection; and $M_f = 11{,}900 \times 1.382 = 16{,}400$ ft-kips. Comparison of the foregoing results for the deflection-theory correction indicates that in the fixed arch the effect of horizontal deflection is of minor importance and ordinarily may be neglected.

To compute deflections:

$$(P_{e-v})' = \frac{27}{35} \times 26{,}000 = 20{,}000 \text{ kips};$$

$$(P_{e-h})' = \frac{384}{205} \times \frac{9.6 \times 29{,}400 \times 11{,}688}{150 \times 950} = 43{,}400 \text{ kips};$$

$$(P_e)' = \frac{20{,}000}{1 + \frac{35}{9}\left(\frac{150}{950}\right)^2} = 18{,}250 \text{ kips}; \quad \eta_e = \frac{11{,}900}{20{,}000} = 0.595 \text{ ft};$$

$$\eta = 1.382 \times 0.595 = 0.82 \text{ ft}; \quad \rho_e = \frac{11{,}900}{43{,}400} = 0.275 \text{ ft};$$

$$\rho = 1.382 \times 0.275 = 0.38 \text{ ft}; \quad u_e = \frac{11{,}900}{18{,}250} = 0.65 \text{ ft};$$

and

$$u = 1.382 \times 0.65 = 0.90 \text{ ft}.$$

The deflection correction would actually be less than the foregoing due to greater stiffness near the skewback than assumed in the formulas. A closer agreement with the percentage of correction of the authors is obtained for a 0.415-span load with a 0.32 L distance between points of contraflexure as shown in Fig. 12 and Table 3.

Half-span wind load moments as obtained by the usual elastic theory, like live-load moments, are subject to correction for the effect of deflection. Such correction is not considered by the authors of the first Symposium paper in writing Eqs. 16, 17, and 21 for permissible flange stress, including the effect of buckling.

Eqs. 17 and 21 are proposed for design use by the authors of the first Symposium paper. When corrected for the effect of deflection on wind moment, they should prove useful in conjunction with the usual computation of maximum stresses for the design loads. For box-girder rib construction the safety factor of approximately 1.8 currently used for (dead + live + impact)-load stresses appears proper for (dead + live + impact)-load + temperature + error stresses in arches, increased for deflection or buckling effect. For

single-web rib construction the working stress should be decreased in proportion to L/b or L/r to allow for the greater tendency toward lateral buckling between bracing points. In conformance with current practice, the safety factor for box-girder ribs would be reduced to 1.5 with the addition of wind stresses, also increased for deflection effect.

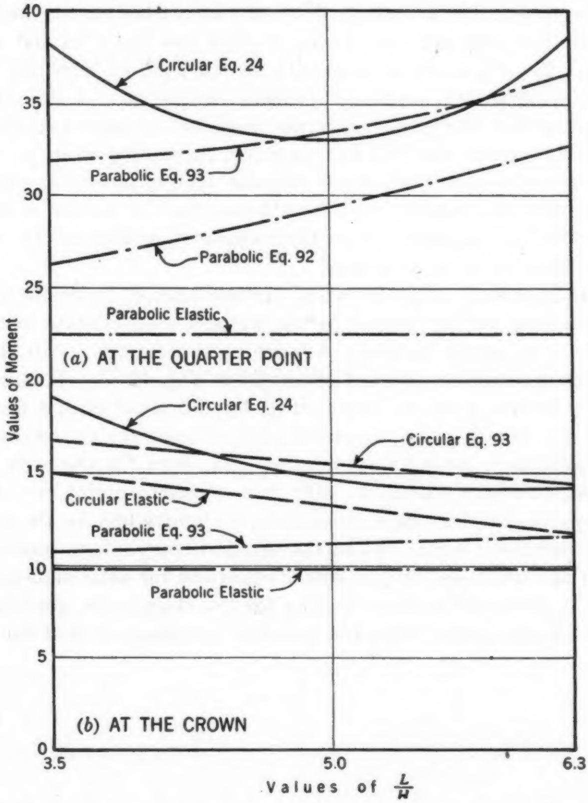


FIG. 42.—COMPARISON OF MOMENTS OF TWO-HINGED ARCH FOR VARYING RISE RATIOS (L = 950 FT; I = 116.14 FT^4 AT CROWN)

It is regrettable that the authors of the second Symposium paper were unable to make a complete determination of the deflection effects on a circular fixed arch, as they have done so excellently for the two-hinged type, or that the findings obtained on the fixed model were not published, unless they were not considered reliable.

The approximate method of the writer for parabolic arches, with moment of inertia varying as the secant of the slope of the rib axis, gives final quarter-point moments for half-span load on two-hinged arches of varying rise ratio which are in reasonably good agreement with those of Eq. 24 for circular arches of constant moment of inertia where the length-rise ratio L/h is 6.3 or 5.0. Although the final positive moment at the quarter point by Eq. 24 for the

Rainbow arch two-hinged design is less for $L/h = 5.0$ than for the flatter arch with $L/h = 6.3$, that for the arch of high rise with $L/h = 3.5$ is practically the same as that for the flattest arch ($L/h = 6.3$). This is contrary to the results for the parabolic arch, for which the final moment, including the effect of deflection, decreases with increase in rise for the range of rise ratios considered. Comparative results are shown in Fig. 42(*a*). Although the method of the writer will be less accurate for arches of high rise than for flat arches, it is probable that the difference in moments for the arch of high rise ($L/h = 3.5$) is due principally to the greater curvature of the axis of the circular arch. With increasing rise the greater average moment of inertia of the parabolic arch of variable section also will have a small, increasing effect in reducing the deflection correction over that of the circular arch of constant section. Thus the writer concludes, contrary to the authors, that for arches of high rise the deflection correction for parabolic or three-centered arches will be considerably smaller than that for circular arches.

Maximum moments at the crown of the two-hinged design for the Rainbow arch as determined by Eq. 24 and by the writer's approximate method for the circular arch of constant moment of inertia are compared with those for the parabolic arch of variable moment of inertia in Fig. 42(*b*). It is important to note that the crown moment, determined by the usual elastic theory, in the circular arch for one-third span central load, is appreciably greater than that in the parabolic arch for 0.3 span central load, even for the arch of flat rise, and that the difference increases with rise. Quarter-point moment by the elastic theory, for the rise range considered, is slightly less for the circular arch than for the parabolic arch. When the elastic-theory crown moments for the circular arch are increased by the writer's method for deflection-theory effect, and using π^2 in place of 9.6 in computing the buckling loads, good agreement is obtained for all rise ratios, with the possible exception of that for $L/h = 3.5$.